AD

TECHNICAL REPORT
NATICK TR-80/036

LEVEL II

(12)

AD A094163

INVESTIGATIONS TO DEFINE ACCEPTABILITY TOLERENCE RANGES IN VARIOUS REGIONS OF COLOR SPACE

by

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Contract Number
DAAK60-78-C-0084

September 1980

UNITED STATES ARMY
NATICK RESEARCH and DEVELOPMENT LABORATORIES
NATICK, MASSACHUSETTS 01760

Clothing, Equipment and Materials
Engineering Laboratory
CEMEL - 225

81 1 23 058

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: NATICK/TR-80/036

2. GOVT ACCESSION NO.: AD-A094 163

3. RECIPIENT'S CATALOG NUMBER:

4. TITLE (and Subtitle): INVESTIGATIONS TO DEFINE ACCEPTABILITY TOLERANCE RANGES IN VARIOUS REGIONS OF COLOR SPACE

5. TYPE OF REPORT & PERIOD COVERED: Contractor - Final

6. PERFORMING ORG. REPORT NUMBER: CEMEL 225

7. AUTHOR(s): Eugene Allen and Barry Yuhas

8. CONTRACT OR GRANT NUMBER(s): DAAK60-78-C-0084

9. PERFORMING ORGANIZATION NAME AND ADDRESS: Lehigh University, Bethlehem, PA 18015

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Materials Testing Technology

11. CONTROLLING OFFICE NAME AND ADDRESS: US Army Natick R&D Laboratories, ATTN: DRDNA-VTC, Natick, MA 01760

12. REPORT DATE: Sep 1980

13. NUMBER OF PAGES: 73

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): NATICK/CEMEL TR-80/036, 225

15. SECURITY CLASS. (of this report): Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report):
Approved for public release;
Distribution is unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):

18. SUPPLEMENTARY NOTES:

19. KEY WORDS (Continue on reverse side if necessary and identify by block number):
Color, Color Measurement, Acceptance, Color Tolerances

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
Current acceptance of goods for color by the Army depends on visual comparison against a standard and eight limit samples. The Army wished to have a numerical method of setting color tolerances to be used with instrumental measurement. The limit samples, on examination, were seen to be somewhat erratic as guides for instrumental tolerances. Instead, we selected pairs of samples from a large number of previous submissions by industry. These pairs showed four graduated lightness steps, four graduated chroma steps, and four graduated hue steps. Six observers looked at each pair 10 times, randomly interspersed with other pairs,

DD FORM 1473, 1 JAN 73 — EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

205450

and issued a pass or fail judgement each time. From these data we established lightness, chroma, and hue tolerance limits. For an olive green and a tan shade, these tolerances were roughly in the ratio 3:2:1; for a dark blue, the ratios were roughly 2:2:1. We wrote simple equations that can be used with instrumental measurement in order to determine quickly whether a sample passes or fails. We recommend similar treatment of all samples for which the Army desires acceptability limits.

PREFACE

This work was performed under Contract Number DAAK 60-78-C-0084 for which Mr. Alvin O. Ramsley and Miss Therese R. Commerford were project officers. We wish to thank them and Mr. Charles R. Williams for the support and guidance they provided.

We greatly appreciate the constructive suggestions made by the Committee on Color Measurement of the National Research Council during the course of this investigation. The committee members are David L. MacAdam, chairman; Michael E. Breton, Ellen C. Carter, Franc Grum, John J. Hanlon, and Robert F. Hoban. We are particularly indebted to Dr. Breton for his recommendation of the method of constant stimuli and the logistic function, and to Dr. MacAdam for his suggestion that ellipsoids be used as a basis for the acceptability equations.

We also wish to thank the observers, who gave of their time to cooperate in making this work a success.

Charles Hunt, Riegel Textile Manufacturing Co., Trion, Ga.
Maurice Larrivee, Natick R&D Command, Natick, Mass.
Carol Neri, Defense Personnel Support Center, Philadelphia, Pa.
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INVESTIGATIONS TO DEFINE ACCEPTABILITY TOLERANCE RANGES IN VARIOUS REGIONS OF COLOR SPACE

I. INTRODUCTION

The establishment of acceptability tolerance limits for purchased goods is a problem as old as the practice of colorimetry. Basing the tolerance on a perceptibility criterion has become discredited, as it is now generally realized that perceptibility and acceptability are not the same. A method of setting up tolerance limits that really represent the informed judgment of experienced inspectors is greatly to be desired. This need becomes all the more cogent as instrumental color measurement gradually displaces visual methods.

The United States Army has been accepting shipments of textiles to be made into army uniforms on a visual basis. The inspector is provided with a standard sample and eight limit samples representing maximum allowable deviations from the standard in each of eight directions in color space. Armed with these nine samples, the inspector examines the submitted sample and issues a pass or fail verdict. This visual method has proved troublesome, as evidenced by frequent disagreements between the Army and the textile mills submitting the samples. The Army therefore wishes to change its acceptance procedures to one using instrumental measurement and valid tolerance limits for acceptance.

The object of this investigation was to develop a procedure for setting up these valid acceptance limits for use with instrumental measurement. The Army submitted the standards and visually selected limit samples for twelve colors of long-term interest. In addition, some 200-300 samples were submitted for each of three other shades. These samples were routine submissions for acceptance or rejection received from various textile mills. We were not requested to set up tolerances for any of these standards; we were rather asked to develop a modus operandi that the Army could use to set up such tolerances for any sample in the future.

II. TESTS ON STANDARD AND LIMIT SAMPLES

A. Relationships between standard and limit samples in color space.

Ideally, the standard and the eight limit samples for any color should form a three-dimensional figure in color space, with the standard somewhere in the interior of the figure and all the limit samples on the boundary. We wished to see if this was indeed the case for limit standards that had been chosen visually and been used successfully for several years in Government procurement.

We obtained measurements of all the standard and limit samples on a Diano-Hardy spectrophotometer, together with calculated CIELAB coordinates L*, a* and b*. We then wrote a computer program that created a three-dimensional figure in CIELAB color space for each of the twelve colors. This figure was constructed

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from the standard and the eight limit samples all taken as a group. The program in effect created all possible tetrahedra that could be constructed from all the nine samples taken four at a time, and then put these tetrahedra together to form the three-dimensional figure The program then determined which of the nine samples (the standard and the eight limit samples) lay inside the figure and which lay on the boundary. Appendix A describes the program and presents the mathematical details, as well as lists of the computer program itself (program RAMSLEY).

Table 1 presents the computer printout from this program. For each of the twelve shades the printout shows the designations of the limit samples, the CIELAB coordinates of the limit samples as well as the standard, and an indication of whether or not each of the standards and limit samples was on the boundary of the three-dimensional figure. We see that in only three of the twelve cases did the standard lie inside the figure; in the other nine cases it was on the boundary. Furthermore, in ten out of the twelve cases one or more of the limit samples fell inside the figure; in only two sets were all the limit samples on the boundary.

Figures 1 and 2 show CIELAB color space plots of the standard and the limit samples for Olive Drab 7, untreated cotton duck, 8.25 oz. Figure 1 shows a plot of b* against a*. The isohue line indicated on the figure is the line joining the standard with the origin; the isochroma line is a segment of a circle drawn through the standard around the origin. The letters shown are abbreviations for the designations of the limit samples; their meaning can be inferred from the designations listed for this sample in Table 1. Although they are useful to visual graders, we see little correlation between the color names applied to some of the limit samples and their position on this diagram. In Figure 2, which is a plot of L* against a*, we see that there is good correlation between the thin or full designation and the lightness of the sample; the thin samples are uniformly lighter than standard and the full samples darker. A glance at the L* values in Table 1, however, shows that this correlation does not strictly hold in all cases; there are cases where the thin samples are darker than standard or the full samples lighter.

Summarizing the results of this section, we see that the standard and the limit samples do not, in many cases, have the expected orientation of standard in the middle and limit samples on the outside. Also, the descriptive designations of the limit samples do not always show a correlation with their plotted points in CIELAB color space. These factors probably reflect the experience of dyers, their practices in matching shades, and their terminology.

B. Determination of minimum ellipsoid containing standard and limit samples.

1. <u>Non-tilting ellipsoid, unconstrained axis angle</u>. In order to learn more about the orientation of the standard and limit samples in CIELAB

Table 1. Designations, CIELAB Coordinates, and Geometric Orientations of Standards and Limit Samples (D75, CIE 1931)

OLIVE DRAB 7, UNTREATED COTTON DUCK, 8.25 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	34.12	-2.71	10.85	ON BOUNDARY
THIN STANDARD	34.45	-2.64	11.07	NOT ON BOUNDARY
THIN YELLOW	36.05	-3.15	11.75	ON BOUNDARY
THIN GREEN	36.25	-2.70	11.04	ON BOUNDARY
THIN RED	36.56	-2.22	10.75	ON BOUNDARY
FULL STANDARD	32.65	-2.34	10.65	ON BOUNDARY
FULL YELLOW	33.19	-2.49	10.84	ON BOUNDARY
FULL GREEN	33.53	-2.73	10.82	ON BOUNDARY
FULL RED	32.63	-1.70	10.13	ON BOUNDARY

TAN 46, COTTON POPLIN, 4 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	68.90	1.03	13.69	ON BOUNDARY
THIN STANDARD	71.13	.69	13.47	ON BOUNDARY
THIN RED	67.88	1.15	13.21	NOT ON BOUNDARY
THIN BLUE	69.57	1.20	12.13	ON BOUNDARY
FULL STANDARD	67.72	.76	13.20	ON BOUNDARY
FULL YELLOW	68.40	.31	13.83	ON BOUNDARY
FULL RED	67.10	1.30	12.86	ON BOUNDARY
FULL BLUE	69.23	.81	13.09	NOT ON BOUNDARY

TAN M-1, CL. POLY/WOOL TROP. 9 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	54.86	2.96	15.73	NOT ON BOUNDARY
THIN STANDARD	55.28	2.77	15.75	NOT ON BOUNDARY
THIN YELLOW	55.99	2.62	15.82	ON BOUNDARY
THIN GREEN	54.67	2.75	15.30	ON BOUNDARY
THIN RED	54.74	3.15	15.62	ON BOUNDARY
FULL STANDARD	54.76	2.85	15.99	NOT ON BOUNDARY
FULL YELLOW	54.42	2.92	16.06	ON BOUNDARY
FULL GREEN	54.65	2.41	15.82	ON BOUNDARY
FULL RED	54.19	3.17	15.85	ON BOUNDARY

ARMY GREEN 44, WL GAB., 13OZ.

SAMPLE	L	A	B	POSITION
STANDARD	26.49	-3.36	1.34	ON BOUNDARY
THIN STANDARD	26.68	-3.39	1.12	ON BOUNDARY
THIN BLUE	26.51	-3.53	1.05	ON BOUNDARY
THIN GREEN	26.20	-3.61	1.07	ON BOUNDARY
THIN RED	26.58	-3.11	1.52	ON BOUNDARY
FULL STANDARD	25.87	-3.15	1.30	ON BOUNDARY
FULL BLUE	25.83	-3.38	.88	ON BOUNDARY
FULL GREEN	26.13	-3.51	1.42	ON BOUNDARY
FULL RED	26.34	-3.09	1.63	ON BOUNDARY

Table 1. (continued)

OLIVE GREEN 107, NYLON/CTN (50/50) POPLIN, 5 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	32.26	-3.46	11.37	NOT ON BOUNDARY
THIN STANDARD	33.59	-3.65	11.93	ON BOUNDARY
THIN YELLOW	33.84	-3.66	11.95	ON BOUNDARY
THIN GREEN	32.95	-3.68	11.32	ON BOUNDARY
THIN RED	33.52	-3.56	11.74	ON BOUNDARY
FULL STANDARD	31.18	-3.21	11.21	ON BOUNDARY
FULL YELLOW	31.89	-3.28	11.73	ON BOUNDARY
FULL GREEN	32.04	-3.65	11.55	ON BOUNDARY
FULL RED	31.95	-3.31	11.32	ON BOUNDARY

BLUE 150TROP. WL. 10 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	15.46	1.12	-4.52	ON BOUNDARY
THIN STANDARD	15.70	1.31	-4.61	ON BOUNDARY
THIN BLUE	15.61	1.23	-4.91	ON BOUNDARY
THIN GREEN	15.67	1.11	-4.50	NOT ON BOUNDARY
THIN RED	15.60	1.24	-4.60	ON BOUNDARY
FULL STANDARD	15.70	1.03	-4.18	ON BOUNDARY
FULL BLUE	15.70	1.04	-4.56	ON BOUNDARY
FULL GREEN	15.41	1.08	-4.24	ON BOUNDARY
FULL RED	15.58	1.20	-4.28	ON BOUNDARY

ARMY GREEN 344, POLYESTER/WOOL GABARDINE 9 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	27.12	-3.52	1.47	ON BOUNDARY
THIN STANDARD	27.13	-3.39	1.40	NOT ON BOUNDARY
THIN BLUE	27.30	-3.42	1.20	ON BOUNDARY
THIN GREEN	27.26	-3.60	1.42	ON BOUNDARY
THIN RED	26.47	-3.47	1.31	NOT ON BOUNDARY
FULL STANDARD	27.27	-3.13	1.34	ON BOUNDARY
FULL BLUE	26.65	-3.50	1.14	ON BOUNDARY
FULL GREEN	26.39	-3.68	1.31	ON BOUNDARY
FULL RED	26.44	-3.19	1.34	ON BOUNDARY

ARMY GREEN 344, POLYESTER/WOOL TROPICAL, 10 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	27.33	-3.29	1.43	ON BOUNDARY
THIN STANDARD	26.63	-3.66	1.12	ON BOUNDARY
THIN BLUE	26.37	-3.36	1.08	NOT ON BOUNDARY
THIN GREEN	27.15	-3.79	1.10	ON BOUNDARY
THIN RED	27.10	-3.13	1.34	ON BOUNDARY
FULL STANDARD	25.35	-3.01	.78	ON BOUNDARY
FULL BLUE	24.98	-3.00	.72	ON BOUNDARY
FULL GREEN	26.07	-3.05	1.00	ON BOUNDARY
FULL RED	25.76	-2.75	1.13	ON BOUNDARY

Table 1. (continued)

ARMY GREEN 44, WL. SERGE 15 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	26.25	-3.18	1.11	ON BOUNDARY
THIN BLUE	26.10	-3.53	1.18	ON BOUNDARY
THIN GREEN	26.21	-3.52	1.17	ON BOUNDARY
THIN RED	26.21	-3.46	1.24	ON BOUNDARY
FULL STANDARD	26.15	-3.43	1.00	ON BOUNDARY
THIN STANDARD	26.24	-3.49	1.22	ON BOUNDARY
FULL BLUE	26.47	-3.46	.97	ON BOUNDARY
FULL GREEN	25.79	-3.35	1.09	ON BOUNDARY
FULL RED	26.22	-3.36	1.13	NOT ON BOUNDARY

OLIVE GREEN 108, WOOL/NYLON, 16 OZ. SHIRTING

SAMPLE	L	A	B	POSITION
STANDARD	29.60	-2.66	11.77	ON BOUNDARY
THIN STANDARD	30.51	-2.88	12.35	ON BOUNDARY
THIN YELLOW	30.35	-2.92	12.58	ON BOUNDARY
THIN GREEN	30.52	-3.41	12.21	ON BOUNDARY
THIN RED	30.19	-2.74	12.06	ON BOUNDARY
FULL STANDARD	28.52	-2.75	11.44	NOT ON BOUNDARY
FULL YELLOW	28.48	-2.72	11.64	ON BOUNDARY
FULL GREEN	29.28	-3.16	11.52	ON BOUNDARY
FULL RED	27.81	-2.41	10.70	ON BOUNDARY

OLIVE GREEN 107, CTN. SATEEN, 8.2 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	31.30	-3.46	10.31	ON BOUNDARY
THIN STANDARD	31.78	-4.11	11.10	ON BOUNDARY
THIN YELLOW	31.78	-3.86	11.58	ON BOUNDARY
THIN RED	31.60	-3.58	11.58	ON BOUNDARY
THIN GREEN	30.64	-4.13	10.84	ON BOUNDARY
FULL STANDARD	30.84	-3.56	10.74	NOT ON BOUNDARY
FULL YELLOW	30.62	-3.14	10.84	ON BOUNDARY
FULL GREEN	30.35	-3.73	10.70	ON BOUNDARY
FULL RED	31.53	-3.04	10.88	ON BOUNDARY

BLUE 151, TROP. WL., 1[illegible]

SAMPLE	L	A	B	POSITION
STANDARD	23.48	4.48	-17.33	NOT ON BOUNDARY
THIN STANDARD	23.85	4.68	-17.47	ON BOUNDARY
THIN BLUE	23.54	4.39	-17.10	ON BOUNDARY
THIN GREEN	24.44	4.65	-18.13	ON BOUNDARY
THIN RED	24.18	4.61	-17.72	ON BOUNDARY
FULL STANDARD	23.07	4.40	-17.07	NOT ON BOUNDARY
FULL BLUE	22.84	4.34	-16.72	[illegible] BOUNDARY
FULL GREEN	23.2[illegible]	4.43	-17.52	ON [illegible]
FULL RED	23.12	4.62	-17.19	ON BOUNDARY

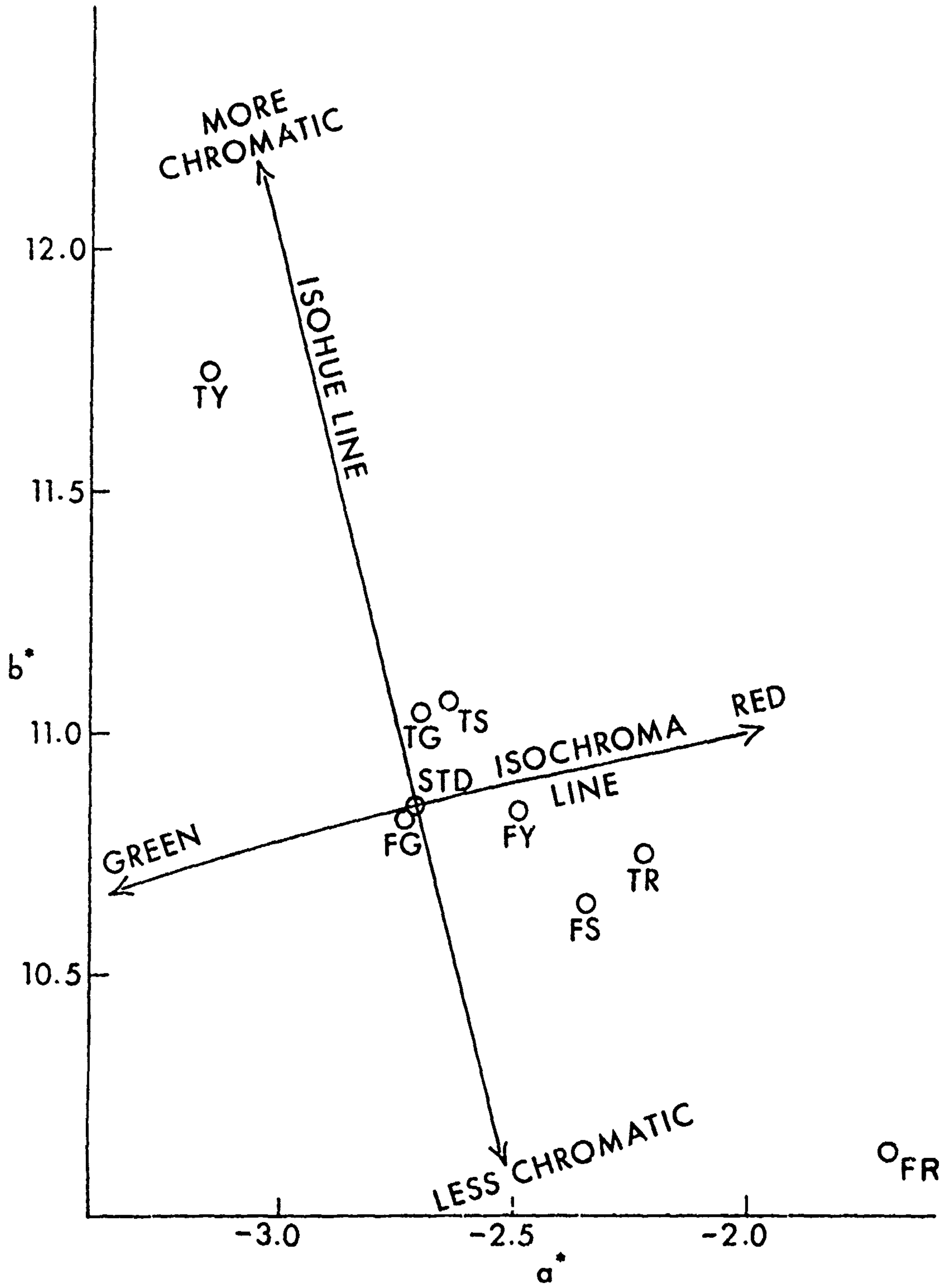

Figure 1. b* vs. a* plot of Standard and limit samples for Olive Drab 7 Untreated cotton duck, 8.25 oz.

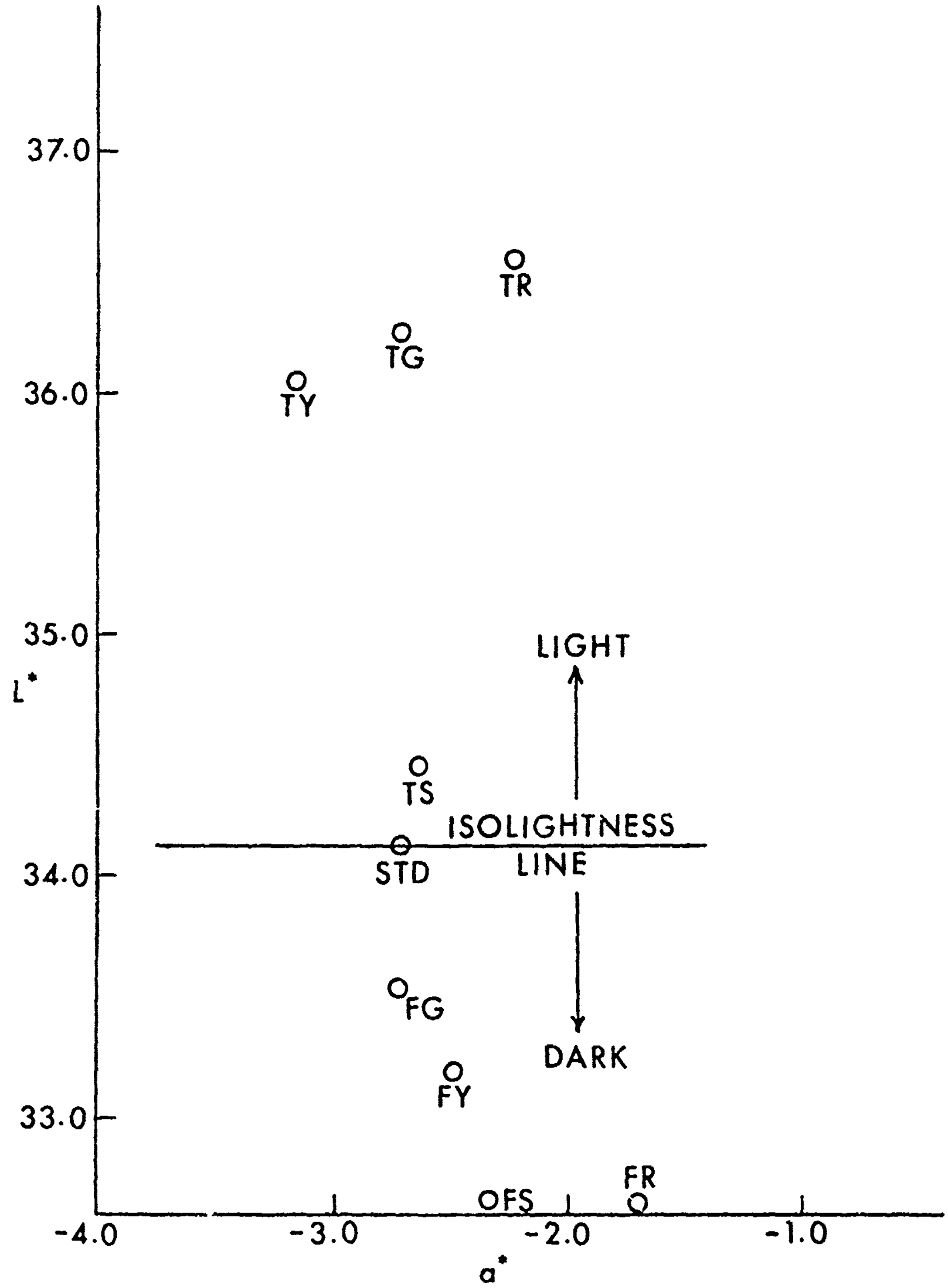

Figure 2. L* vs. a* plot of Standard and limit samples for Olive Drab 7 untreated cotton duck, 8.25 oz.

color space, we wrote a computer program that determined the minimum non-tilting ellipsoid that could be constructed around these samples. The non-tilting feature meant that the ellipsoid had a horizontal major axis, a horizontal minor axis, both in the chromaticity plane, and a vertical axis; the two horizontal axes lay entirely in the a*-b* plane and the vertical axis was perpendicular to the other two. We propose this non-tilting restriction in the spirit of the observations of Schultze and Gall (1) and of MacAdam (2), as applied to perceptibility ellipsoids.

Each ellipsoid was therefore defined by seven parameters: the three position coordinates of the center, the length of the three semiaxes, and the angle that the major axis in the chromaticity plane makes with the a* axis. The computer program varied these seven parameters one at a time in a systematic fashion, each time determining whether a smaller ellipsoid could be constructed that contained all the points for the standard and limit samples. The program stopped when no parameter variation was possible that would shrink the ellipsoid without excluding one of the points. Appendix B describes the mathematics used and presents the computer program, called ELPSOID. The results from this program are given in Table 2. In this table the 12 colors are presented in the same order as they are in Table 1.

2. <u>Non-tilting ellipsoid, axis angle equal to hue angle</u>. We wrote another minimum ellipsoid program, similar to the first except that this time we had only six adjustable parameters instead of seven. The axis angle was set equal to the hue angle; the latter was the angle between the line connecting the center of the ellipsoid with the L* axis and the a* axis. Thus, one of the horizontal axes of the ellipsoid always coincides with the isohue direction, the other horizontal axis with the isochroma direction. Figure 3 serves as an illustration, in two-dimensional space, of the difference between the two programs. The color points shown were plotted arbitrarily. The two ellipses drawn on the figure, however, were calculated by a variation of the two programs that constructs ellipses instead of ellipsoids. The dashed ellipse is the smallest ellipse of any orientation that can be drawn surrounding all the points whose major axis coincides with an isohue line.

The reason for this second minimum ellipsoid program was that we wished to take advantage of the commercial experience which has been built up in the

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1. W. Schultze and L. Gall, "Experimentelle Überprüfung Mehrerer Farbabstandsformelu bezüglich du Helligkeits - und Sättigungsdifferenzen bei gesättigten Farben," Farbe 18, 131-148 (1969).

2. D. L. MacAdam, in Color Metrics, edited by J. J. Vos, L.F.C. Friele and P.L. Walraven (AIC/Holland, Soesterburg, 1972) pp 160-170.

Table 2. Minimum Ellipsoid Characteristics

	COLOR NO.											
	1	2	3	4	5	6	7	8	9	10	11	12
<u>Unconstrained Azimuthal Angle</u>												
Ellipsoid Center: L^*	34.44	68.91	54.89	26.25	32.51	15.60	26.84	26.27	26.24	29.11	31.23	23.64
a^*	-2.42	.77	2.82	-3.33	-3.43	1.13	-3.41	-3.27	-3.36	-2.86	-3.66	4.49
b^*	10.84	12.98	15.73	1.18	11.58	-[illegible].44	1.31	1.01	1.10	11.74	10.97	-17.42
Semiaxis Lengths: Maj.Horiz.	1.58	.99	.44	.53	.55	.48	.39	.61	.19	1.35	.78	1.17
Min.Horiz.	.26	.65	.43	.27	.36	.17	.19	.50	.19	.47	.60	.19
Vertical	2.56	2.62	1.29	.55	2.15	.22	.63	1.78	.45	2.40	.99	1.05
< between Horiz.axis & a^* axis,deg.	-48.2	-62.0	73.6	68.8	-58.7	-76.4	1.8	-23.3	<u>a</u>	-65.8	-55.7	-80.4
Vol.	4.40	7.07	1.03	.33	1.78	.08	.19	2.28	.07	6.36	1.95	.98
<u>Constrained Azimuthal Angle</u>												
Ellipsoid Center: L^*	34.48	68.79	54.81	26.22	32.44	15.60	26.89	26.40	26.21	29.47	31.16	23.65
a^*	-2.42	.91	2.79	-3.35	-3.50	1.15	-3.43	-3.23	-3.37	-2.85	-3.63	4.51
b^*	10.86	13.18	15.77	1.26	-1.47	-4.49	1.33	1.03	1.07	11.41	10.96	-17.36
Semiaxis Lengths: Maj.Horiz.	1.55	1.27	.48	.21<u>b</u>	.66	.44	.45	.63	.22	1.54	.91	1.06
Min.Horiz.	.93	.74	.43	.60<u>b</u>	.31	.15	.22	.43	.17	.52	.62	.16
Vertical	3.15	2.54	1.30	.63	2.21	.24	.69	2.04	.43	2.37	.90	1.37
< between Horiz.axis & a^* axis,deg.	-77.4	86.1	80.0	-20.6	-73.0	-75.6	-21.1	-17.7	-17.7	-76.0	-71.7	-75.4
Vol.	19.04	9.96	1.12	.34	1.86	.07	.28	2.32	.07	7.94	2.14	.99

<u>a</u> Two horizontal axes equal; therefore angle cannot be specified.

<u>b</u> Iso(hue-lightness) axis (considered by computer program to be major horizontal axis) is shorter than iso(chroma-lightness) axis.

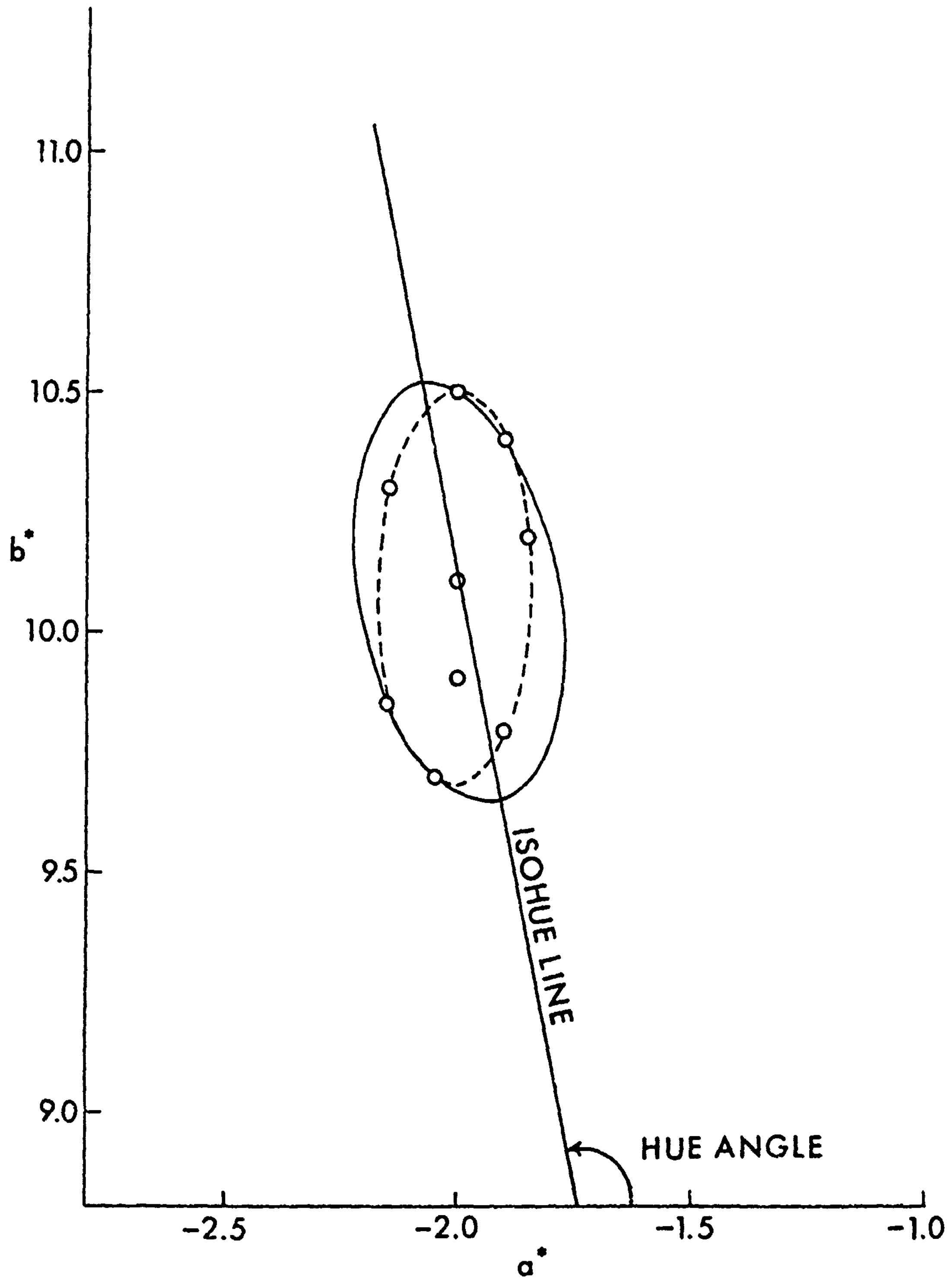

Figure 3. Illustration in two diminsions of the two minimum ellipsoid.

textile industry relating to acceptance of production lots. This experience has indicated that acceptability tolerances in a given uniform color space should be so arranged as to allow the greatest leeway in the lightness direction, intermediate latitude in the chroma direction, and the most stringent tolerances in the hue direction (3). We can thus picture an acceptability ellipsoid with the largest axis pointing in the L* direction, the intermediate axis in the chromaticity plane along an isohue line, and the shortest axis in the chromaticity plane perpendicular to the isohue line. The ratios between these three tolerances have typically been found to range between 3:2:1 and 4:?:1, in CIELAB space.

This computer program was called ELPSOD2, and is given in Appendix B.

Results of this program are presented in Table 2.

C. Results of ellipsoid tests; hue-chroma-value axis ratios.

Comparison of the axis angles given by the two minimum ellipsoid programs is interesting and is shown in Table 3, which was abstracted from the results in Table 2. We see a remarkable agreement between the two programs with respect to the axis angle of the ellipsoid. In six out of eleven cases the angles agreed to 10^{o} or less; in eight out of eleven cases the agreement is to 20^{o} or less. Since in one of the programs, one horizontal axis was constrained to the isohue direction, whereas in the other program the axes were unconstrained, we conclude that the acceptability ellipsoids, at least insofar as they are defined by the standard and limit samples, naturally tend to orient themselves in the isohue and isochroma directions.

Table 4, calculated from the figures in Table 2, shows the ratios of the three principal axes for the constrained ellipsoids, as well as the ellipsoid volumes. We see that in nine out of the twelve cases we have the expected size relationship of the largest axis being in the lightness direction, the intermediate axis in the chroma direction, and the smallest axis in the hue direction. In one of the other three cases the lightness and chroma axes were equal; in the second of the three exceptions the chroma and hue axes were reversed from the expected order, and in the third case the lightness and chroma axes were reversed. These results seem to bear out the commercial experience for textile industry color acceptances mentioned above.

III. TESTS ON MULTIPLE SAMPLE COLLECTIONS

A. Selection of sample pairs

As a general policy, we thought that acceptability standards should be based on the visual judgment of trained and experienced observers. In view of the ellipsoid orientation indicated by the work just described, we thought that it would be appropriate to establish six tolerances: two in the plus and minus

3. "The Hue Angle Sort Program," pamphlet issued by Hunter Associates Laboratory, Inc.

TABLE 3. ANGLE OF PRINCIPAL HORIZONTAL AXIS OF ELLIPSOID WITH a* COORDINATE AXIS, IN DEGREES.

	Angle unconstrained	Angle constrained along isohue line	Corrected Difference
Olive Drab 7, Cotton Duck	-48	-77	29
Olive Green 107, Nylon/cotton	-59	-73	14
Olive Green 108, Wool/nylon	-66	-76	10
Olive Green 107, Cotton sateen	-56	-72	16
Army Green 44, Wool Gabardine	69	-21a	0
Army Green 344, Polyester/Wool Gabardine	2	-21	23
Army Green 344, Tropical Wool	-23	-18	5
Army Green 44, Wool Serge	b	-18	
Tan 46, Cotton Poplin	-62	86	32
Tan M-1, Polyester/Wool Tropical	74	80	6
Blue 150, Tropical	-76	-76	0
Blue 151, Tropical	-80	-75	5

a Axis in isohue direction smaller than axis in isochroma direction; angle must therefore be increased by 90° before comparison with value in left column.

b. Two horizontal axes equal; therefore angle cannot be specified.

TABLE 4. AXIS RATIOS AND ELLIPSOID VOLUMES FOR ELLIPSOIDS POINTING IN ISOHUE DIRECTION

	Lightness:Chroma:Hue	Ellipsoid Volume
Olive Drab 7, Cotton Duck	3:37 : 1.66 : 1.00	19.04
Olive Green 107, Nylon/Cotton	7.23 : 2.16 : 1.00	1.86
Olive Green 108, Wool/Nylon	4.59 : 2.98 : 1.00	7.94
Olive Green 107, Cotton Sateen	1.45 : 1.45 : 1.00	2.14
Army Green 44, Wool Gabardine	1.05 : 0.36 : 1.00	0.34
Army Green 344, Polyester/Wool	3.23 : 2.10 : 1.00	0.28
Army Green 344, Tropical	4.72 : 1.45 : 1.00	2.32
Army Green 44, Wool Serge	2.56 : 1.29 : 1.00	0.07
Tan 46, Cotton Poplin	3.44 : 1.72 : 1.00	9.96
Tan M-1, Polyester/Wool Tropical	3.00 : 1.10 : 1.00	1.12
Blue 150, Tropical	1.62 : 3.01 : 1.00	0.07
Blue 151, Tropical	8.41 : 6.53 : 1.00	0.99

chroma directions, two in the plus and minus lightness directions, and two in the plus and minus hue directions. The tolerances would be set up on the basis of the responses of the inspectors to samples shown to them. We would then establish an acceptability ellipsoid that incorporates these tolerances and that has its three axes oriented in the hue, chroma, and lightness directions.

The problem then arises as to how to obtain samples that differ from the standard only in hue, chroma, or lightness and in varying amounts for each. We rejected a preliminary plan involving the dyeing of samples of various degrees of color difference in each of the six directions when we realized how difficult it would be to prepare such dyeings. In order to obtain samples having the precise location in color space that we want, we would have to have at least two and probably more trial dyeings for each sample. Such a procedure would take an inordinate amount of time.

A much better plan seemed to be to try to find the desired samples among previous submissions of samples from textile mills. For each shade that is actively used by the Army, textile mills are continually submitting samples that are either accepted or rejected by the inspectors. Fortunately, a set of some 200 to 300 samples was available for each of three shades of interest to the Army: olive green, tan, and dark blue. These samples are located in a cloud of points close to the standard, since they are intended to match the standard. It should be possible to find, among all these samples, test cases that could be shown to the observers and that would serve to define the required tolerances.

We wished to find samples representing four color differences of increasing amount in each of the six directions mentioned above. These color differences would ideally extend from completely acceptable (for the smallest color difference) to completely unacceptable (for the largest color difference). The two intermediate color differences would presumably be close to the tolerance limits. The procedure would be to show each of the color differences to an observer ten times (randomly mixed with other samples so that the inspector does not realize that he is looking at the same sample repeatedly). Each time that the observer is shown the color difference, he is asked whether he would accept or reject such a difference if it were an incoming shipment against standard. The experimental regime is known as the method of constant stimuli.

To explain the procedure, let us assume that the four color differences chosen are 0.5, 1.0, 1.5, and 2.0 CIELAB units. Let us observe that the observer accepts the 0.5 color difference ten times out of ten; the 1.0 color difference seven times out of ten; the 1.5 color difference three times out of ten. It can then easily be seen that the 50% acceptance limit should be close to 1.25 color difference units. In fact, in the more general case, the method of logistic functions (see below) is used to determine the 50% acceptability limit.

The question then arises as to how to select these color differences out of the cloud of 200 to 300 samples. We realize that it is not necessary to have only a single standard against which all the judgments would be made. All that is necessary is a selection of sample pairs, of which one would play the role of standard and the other would play the role of sample submitted for acceptance. Since we have six directions in color space with four degrees of color difference for each direction, we need 24 sample pairs. These pairs were chosen in the following way:

We first measured all the samples for each of the three colors on the Diano-Hardy spectrophotometer, and obtained the CIELAB coordinates (D75, CIE 1931). We then created a computer file with an entry for each of the 200 to 300 samples for each color; the file contained the numerical sample designations and the three CIELAB coordinates L*a*b*. We rearranged this file so that it was in increasing numerical order for the sample designations. This rearrangement was carried out by Program REARRAN, given in Appendix C.

We then wrote a computer program which we called Program SIXWAYS. For each of the three collections of samples, the program took each individual sample, one at a time, and constructed three lines in CIELAB color space passing through the sample. The first line was an iso (hue-lightness) line, which means that it was a line in the a* - b* plane that joined the origin with the point in question. Since the line passed through the point for the sample, it extended from the sample in two directions: the direction of increasing chroma and the direction of decreasing chroma. The second line was an iso (chroma-lightness) line, or a line in the a* - b* plane that was perpendicular to the first. This line also passed through the sample and extended from the sample in two opposite hue directions. The third line was an iso (hue-chroma) line, or a vertical line through the sample perpendicular to the a* - b* plane and extending from the sample in a plus lightness and a minus lightness direction.

The computer program thus contained an outer loop that was traversed n times, where n is the number of samples in the collection. For each of these samples, the program constructed the six lines in CIELAB color space just described. The program then went through an inner loop, with n - 1 passages, during which it again considered every sample in the collection and determined whether it lay on one of the six lines that was constructed (the program actually determined whether the sample deviated from these lines by no more than 0.1 CIELAB unit). If the sample did so lie, the program calculated the color difference between the two samples. We thus had, for each of the samples in the collection considered temporarily as a standard, a subset of samples extending from the standard in each of the six directions, together with the associated color differences between each sample in the subset and the temporary standard. It was therefore relatively easy to select pairs of samples for which the "sample" deviated from the "standard" in each of six directions and with four different color differences in each of the directions. Notice that there was no single fixed standard chosen; the standard varies with each pair. Notice also that the sizes of the color differences available were determined and limited by the particular samples in the collection.

Appendix D shows the mathematics that was used to determine how far a given sample deviated from the six iso lines radiating from another sample. The same appendix presents the listing of programs SIXWAYS. Also included in the appendix is a typical page from the output of program SIXWAYS.

Figure 4 shows an a* - b* plot of the samples making up the four color differences of increasing chroma and the four color differences of increasing hue for the olive green collection. The points represent actual samples in the collection; the solid lines join pairs of points for the various color differences. The dashed lines are the true constant chroma lines and constant hue lines, and it can be seen that in no case does a sample deviate from these lines by more than 0.1 CIELAB unit.

We were able to physically reduce the number of pairs of samples chosen by almost half, because in all but a few cases, the same pair of samples was used for a minus difference as for a plus difference. For example, let us suppose that Samples A and B have the same hue and lightness but differ by 1.0 CIELAB unit in chroma. In order to determine whether there is a difference in acceptability in the plus chroma or the minus chroma direction, the pair will be shown to the inspector in two different ways; in the first way, Sample A is the standard and Sample B is the sample to be judged against the standard; in the second way, Sample B is the standard and Sample A is the sample. In fact, if Sample B would appear under the Sample A heading in the printout from program SIXWAYS as being more chromatic than Sample A, then Sample A would appear under the Sample B heading as being less chromatic.

B. Presentation of Sample Pairs

As a result of work described in the preceding section, we had 72 sample pairs to show to the observers (three shades, six directions in color space for each shade, that is $\pm$ chroma, $\pm$ lightness, and four degrees of color difference for each direction). We wished to show each sample pair to each observer ten times, making a total of 720 individual observations by each inspector. In order not to confuse the observers more than was absolutely necessary, we did one shade at a time, involving 240 judgments per shade. We wrote a computer program, entitled Program FUNFER, that used a random number generator to randomize the order of presentation of these 240 trials (see Appendix E). The observers were not told that there were replicate sample presentations, although they probably guessed that from the fact that there were many fewer sample pairs than there were presentation. In order to prevent the inspector being influenced by the location of the sample pair as he saw it being removed, we had a "lazy susan" sample holder constructed, on which the samples were arranged circularly and accessed by rotating a tray In this way, distinctive sample positions were abolished.

We administered the test to six observers, three from the government service and three from industry. In addition, a seventh observer, previously from industry but now in the government service, looked at only one of the three shades--the dark blue. The observer was presented with a sample pair

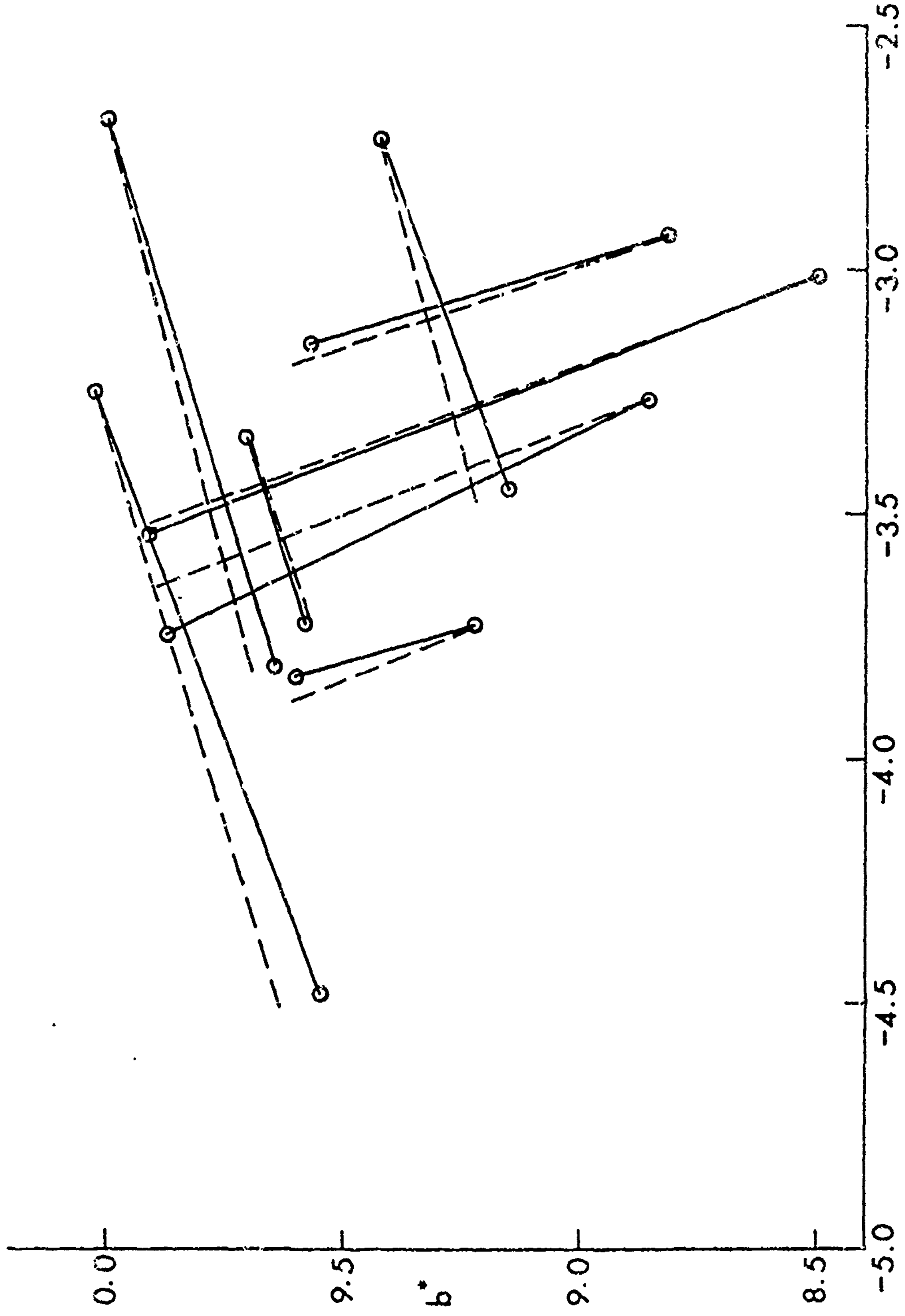

Figure 4. Sample pairs for Olive Green selected for observers' evaluation plotted on a* - b* diagram.

and was told which was the standard and which was the sample to be compared with standard. He or she was told to either accept or reject the sample against standard; no intermediate judgments were allowed. The observers' judgments were tabulated, and the percentage of passes for each pair was noted.

Tables 5, 6 and 7 show the raw results of the observers for all the samples. In the leftmost column, the CIELAB total color difference for each pair is given. In the columns to the right the percent pass judgments for each pair is shown. A, B and C were the three observers from the government service; D, E, and F were the three observers from industry. G was the observer who was previously in in industry and is now in the government service, who looked at only the dark blue series of samples.

We see that in most cases the percentage of samples passed decreases with increasing color difference. Occasionally there are reversals, as indicated by a larger color difference giving a greater percentage of acceptances. In some cases these reversals are severe enough to invalidate that particular set of judgments, as will be seen later. For the most part, however, the results of these tests are surprisingly good.

C. Scoring by Logistic Function

The raw data in Tables 5, 6 and 7 take the form of percent acceptance as a function of color difference. We now have to calculate from these data, on a valid statistical basis, the color difference that would correspond to a 50% acceptance rate. We used the method of logistic functions, explained in Appendix F. This method not only gives the 50% point but also gives some idea of the precision of this determination in the form of a standard deviation.

In Tables 5, 6 and 7 we see next to each group of four acceptance percentages the calculated color difference corresponding to a 50% acceptance rate, together with the standard deviation of this color difference. This standard deviation is really a measure of the self-consistency of the acceptance judgments. If there are serious reversals in the judgments, the standard deviation may be quite large in comparison with the value itself (in some cases larger).

We decided, quite pragmatically, to accept only those values that were at least about four times as large as their standard deviations. In addition, in view of the fact that observer F accepted 18 out of the 24 tan pairs and 20 out of the 24 dark blue pairs on a 100% basis, we considered him unfamiliar with colors and used his results only for the olive green pairs.

D. Results of Tests

Table 8 summarizes the tolerance results that were accepted. The standard deviations shown refer to the values given in this table and do not involve the standard deviations shown in Tables 5, 6 and 7. The following observations can be made:

Table 5. Raw Results for Observers A, B, and C, Together with 50% Acceptance Limits

COLOR: OLIVE GREEN

DELTA E	A PCT. PASS	A BOUNDARY ± STD. DEV.	B PCT. PASS	B BOUNDARY ± STD. DEV.	C PCT. PASS	C BOUNDARY ± STD. DEV.
PLUS CHROMA						
.40	100		100		100	
.80	40		50		70	
1.13	20	.78 ± .11	80	1.31 ± .46	90	4.02 ± 7.01
1.52	20		40		70	
MINUS CHROMA						
.40	100		100		100	
.80	100		50		80	
1.13	100	0.00 ± 0.00	70	1.05 ± .19	80	1.47 ± .38
1.52	100		20		40	
PLUS HUE						
.40	100		70		100	
.77	70		0		40	
1.17	10	.83 ± .08	0	.46 ± .07	0	.71 ± .08
1.32	0		0		0	
MINUS HUE						
.40	40		90		90	
.77	20		0		0	
1.17	0	.34 ± .16	0	.53 ± .07	0	.53 ± .07
1.45	0		0		0	
PLUS LIGHT						
.60	100		90		100	
1.17	70		100		100	
1.82	30	1.76 ± .38	10	1.64 ± .34	70	6.70 ±13.73
2.29	50		40		90	
MINUS LIGHT						
.60	100		100		100	
1.17	100		100		100	
1.82	100	4.46 ± 3.11	10	1.78 ± .29	100	2.29 ± .15
2.29	90		40		50	

Table 5. (continued)

COLOR: TAN

DELTA E	A PCT. PASS	A BOUNDARY ± STD. DEV.	B PCT. PASS	B BOUNDARY ± STD. DEV.	C PCT. PASS	C BOUNDARY ± STD. DEV.
PLUS CHROMA						
.50	80		100		80	
.92	80		40		80	
1.33	10	.95 ± .15	0	.85 ± .09	20	.98 ± .15
1.92	10		0		10	
MINUS CHROMA						
.50	90		100		100	
.92	20		20		30	
1.33	0	.73 ± .10	10	.77 ± .13	0	.80 ± .10
1.92	0		0		0	
PLUS HUE						
.25	40		70		60	
.47	40		60		30	
.69	0	.21 ± .17	0	.43 ± .10	0	.30 ± .08
1.01	0		0		0	
MINUS HUE						
.25	50		70		60	
.48	10		0		70	
.69	0	.25 ± .07	0	.29 ± .04	10	.42 ± .15
1.01	0		0		0	
PLUS LIGHT						
.66	60		100		100	
1.32	30		50		20	
1.95	20	.84 ± .25	0	1.27 ± .14	10	1.09 ± .18
2.65	0		0		0	
MINUS LIGHT						
.66	100		100		100	
1.32	50		20		30	
1.95	30	1.41 ± .17	70	1.50 ± .38	10	1.15 . .16
2.65	10		0		0	

Table 5. (continued)

COLOR: DARK BLUE

DELTA E	A PCT. PASS	A BOUNDARY ± STD. DEV.	B PCT. PASS	B BOUNDARY ± STD. DEV.	C PCT. PASS	C BOUNDARY ± STD. DEV.
PLUS CHROMA						
.36	70		90		80	
.69	20		80		80	
1.07	0	.47 ± .14	0	1.77 ± 1.17	10	.93 ± .23
1.42	20		70		40	
MINUS CHROMA						
.36	80		100		40	
.69	30		90		80	
1.07	10	.59 ± .15	100	6.64 ±14.74	10	.42 ± .22
1.42	30		80		10	
PLUS HUE						
.18	70		100		100	
.38	80		100		90	
.57	60	1.12 ± 1.29	100	1.67 ± 1.48	80	.69 ± .20
.75	50		90		0	
MINUS HUE						
.18	70		100		80	
.38	50		90		90	
.57	100	85.62 ±*1.01	100	.61 ± .14	100	.90 ± .68
.75	60		10		0	
PLUS LIGHT						
.50	50		100		60	
1.00	30		100		10	
1.50	0	.53 ± .20	10	1.29 ± .12	0	.56 ± .13
2.01	0		0		0	
MINUS LIGHT						
.50	90		100		70	
1.00	70		80		10	
1.50	10	1.04 ± .13	20	1.17 ± .13	0	.62 ± .12
2.01	0		0		0	

Table 6. Raw Results for Observers D, E, and F, Together with 50% Acceptance Limits

COLOR: OLIVE GREEN

DELTA E	D PCT. PASS	D BOUNDARY ± STD. DEV.	E PCT. PASS	E BOUNDARY ± STD. DEV.	F PCT. PASS	F BOUNDARY ± STD. DEV.
PLUS CHROMA						
.40	100		100		100	
.80	90		90		100	
1.13	50	1.38 ± .32	40	1.13 ± .17	100	1.42 ± .10
1.52	50		30		30	
MINUS CHROMA						
.40	100		100		100	
.80	40		60		100	
1.13	100	.92 ± .18	20	.85 ± .09	70	1.21 ± .11
1.52	20		10		10	
PLUS HUE						
.40	80		100		100	
.77	0		0		70	
1.17	0	.49 ± .07	0	.55 ± .08	10	.83 ± .08
1.32	0		0		0	
MINUS HUE						
.40	100		100		100	
.77	0		0		10	
1.17	0	.55 ± .08	0	.55 ± .08	0	.60 ± .10
1.45	0		0		0	
PLUS LIGHT						
.60	100		100		100	
1.17	100		90		100	
1.82	0	1.46 ± .12	30	1.50 ± .18	40	3.15 ± 3.52
2.29	0		0		90	
MINUS LIGHT						
.60	100		100		100	
1.17	100		90		100	
1.82	0	1.58 ± .17	20	1.42 ± .16	60	4.97 ± 8.13
2.29	10		0		90	

Table 6. (continued)

COLOR: TAN

DELTA E	D PCT. PASS	D BOUNDARY ± STD. DEV.	E PCT. PASS	E BOUNDARY ± STD. DEV.	F PCT. PASS	F BOUNDARY ± STD. DEV.
PLUS CHROMA						
.50	100		100		100	
.92	0		90		100	
1.33	100	1.04 ± .26	100	*667.20 ±*5.07	80	*378.07 ±*3.62
1.92	0		100		100	
MINUS CHROMA						
.50	100		100		100	
.92	0		100		100	
1.33	70	1.22 ± .28	100	1.92 ± .20	100	5.60 ± 7.94
1.92	0		50		90	
PLUS HUE						
.25	100		100		100	
.47	90		90		100	
.69	0	.54 ± .08	20	.57 ± .07	100	*076.48 ±*8.40
1.01	0		0		100	
MINUS HUE						
.25	100		100		10	
.48	100		90		100	
.69	10	.59 ± .04	10	.55 ± .07	20	.65 ± .18
1.01	0		0		100	
PLUS LIGHT						
.66	100		100		100	
1.32	20		100		100	
1.95	0	1.07 ± .16	100	2.55 ± .20	100	2.47 ± .17
2.65	0		40		30	
MINUS LIGHT						
.66	100		100		100	
1.32	0		90		100	
1.95	0	.93 ± .14	100	2.64 ± .71	100	6.47 ± 6.89
2.65	0		40		90	

Table 6. (continued)

COLOR: DARK BLUE

DELTA E	D PCT. PASS	D BOUNDARY ± STD. DEV.	E PCT. PASS	E BOUNDARY ± STD. DEV.	F PCT. PASS	F BOUNDARY ± STD. DEV.
PLUS CHROMA						
.36	10		100		100	
.69	100		100		100	
1.07	0	2.71 ± 7.00	0	1.41 ± .53	100	*088.70 ±*0.32
1.42	0		60		100	
MINUS CHROMA						
.36	50		100		100	
.69	100		100		100	
1.07	10	.43 ± .16	0	1.24 ± .32	100	*088.70 ±*0.32
1.42	0		50		100	
PLUS HUE						
.18	90		100		100	
.38	100		100		100	
.57	100	.72 ± .31	100	.65 ± .04	100	*472.55 ±*2.88
.75	0		0		100	
MINUS HUE						
.18	100		100		90	
.38	100		100		100	
.57	100	.67 ± .04	100	.67 ± .04	100	21.81 ±*0.04
.75	10		10		80	
PLUS LIGHT						
.50	100		100		100	
1.00	10		100		100	
1.50	10	.80 ± .15	0	1.22 ± .09	100	1.77 ± .12
2.01	0		0		10	
MINUS LIGHT						
.50	100		90		100	
1.00	10		90		100	
1.50	0	.77 ± .13	0	1.15 ± .23	100	1.74 ± .12
2.01	0		0		0	

Table 7. Raw Results for Observer G on Dark Blue, Together with Calculated 50% Acceptance Limits

PCT. PASS	BOUNDARY & STD. DEV.
80 100 10 40	.91 ± .24
100 90 0 100	1.82 ± 1.45
100 100 90 100	*443.87 ±*5.12
100 90 100 30	.68 ± .16
90 90 20 10	1.15 ± .16
100 100 60 10	1.55 ± .14

Table 8. Summary of Tolerance Results

Olive Green

	+ Chroma	- Chroma	+ Hue	- Hue	+ Lightness	- Lightness
A	0.78		0.83		1.76	
B		1.05	0.46	0.53	1.64	1.78
C		1.47	0.71	0.53		2.29
D	1.38	0.92	0.49	0.55	1.46	1.58
E	1.13	0.85	0.55	0.55	1.50	1.42
F	1.42	1.21	0.83	0.60		
Av. ± St. D.	1.13 ± 0.26		0.60 ± 0.13		1.68 ± 0.28	

Tan

	+ Chroma	- Chroma	+ Hue	- Hue	+ Lightness	- Lightness
A	0.95	0.73		0.25		1.41
B	0.85	0.77	0.43	0.29	1.27	1.50
C	0.98	0.80	0.30		1.09	1.15
D	1.04	1.22	0.54	0.59	1.07	0.93
E		1.92	0.57	0.55	2.55	2.64
Av. ± St. D.	1.03 ± 0.37		0.44 ± 0.14		1.51 ± 0.64	

Dark Blue

	+ Chroma	- Chroma	+ Hue	- Hue	+ Lightness	- Lightness
A		.59				1.04
B				.61	1.29	1.17
C	.93				.56	.62
D				.67	.80	.77
E		1.24	.65	.67	1.22	1.15
G	.91			.68	1.15	1.55
Av. ± St. D.	0.92 ± 0.27		0.66 ± 0.03		1.03 ± 0.31	

1. Differences between plus and minus directions. There is no significant difference between the plus hue and minus hue tolerances or between the plus lightness and minus lightness tolerances. The difference between the plus chroma and minus chroma tolerances, however, is debatable. If we apply the Student's t test to the unpaired data, there is no significant difference between plus and minus chroma. If, however, we pair the plus chroma and minus chroma data by observer, we see that three observers had a larger plus chroma tolerance than a minus chroma tolerance for the olive green, whereas no observer had the reverse. Application of the Student's t test to the paired data showed significance for this color. But for the tan, even the paired data did not show a significant difference; three observers favored plus chroma and one favored minus chroma. We decided to consider that no difference existed between any plus and minus directions. We therefore averaged all the values, plus and minus alike in calculating averages and standard deviations, and these latter are included in Table 8.

2. Differences Between Hue, Chroma, and Lightness; Axis Ratios. There is strong confirmation of the 3:2:1 lightness: chroma: hue tolerance ratios in the cases of the olive green and the tan. For the olive green, the ratios are 2.8:1.9:1.0; for the tan, they are 3.4:2.3:1.0. But for the dark blue, the ratios are 1.6:1.4:1.0; although the differences between lightness and hue or between chroma and hue are significant, the difference between lightness and chroma is not. We see that it is dangerous to generalize, and that we must work out individual tolerances for each color.

The absolute values for the hue tolerances lie somewhere in the neighborhood of half a CIELAB unit; those for chroma tolerances are near one CIELAB unit. The lightness tolerances are near one and one-half CIELAB units for the olive green and the tan, and near one CIELAB unit for the dark blue.

3. Differences between Observers from Industry and from Government Service. We might possibly expect that the observers from industry would permit larger tolerances than those from government service. Table 8 shows that this is not the case for the values shown therein; there is no significant difference between the results of the observers from industry and those of the government observers, both taken as a group. An exception is Observer G on the tan and the dark blue, but his results for these colors were not included in Table 8.

IV. ACCEPTABILITY EQUATIONS

We use the tolerances for chroma, hue and lightness given in Table 8 to construct an acceptability ellipsoid for each color. The equation of this ellipsoid determines whether a given sample does or does not pass.

This equation takes the following form:

$$\Delta A = \left[g_{11} \left(\Delta a^*\right)^2 + 2g_{12}\, \Delta a^* \,\Delta b^* + g_{22}\left(\Delta b^*\right)^2 + g_{33}\left(\Delta L^*\right)^2 \right]^{\frac{1}{2}}, \tag{1}$$

where ΔA is an acceptability figure, so scaled that normally if ΔA is less than 1, the sample is accepted; if ΔA is greater than 1, it is rejected (see further discussion on this point below). The quantities Δa^*, Δb^*, ΔL^* are sample minus standard in CIELAB coordinates.

Appendix G explains the derivation and meaning of the acceptability equation. The coefficients g_{11}, $2g_{12}$, g_{22} and g_{33} are given by the following equations where a^*_0 and b^*_0 represent the CIELAB a* and b* values of the standard, c is the chroma tolerance, h is the hue tolerance, and v (for value) is the lightness tolerance:

$$\theta = \tan^{-1}\left(b^*_0 / a^*_0\right), \tag{2}$$

$$g_{11} = \left(\cos^2\theta / c^2\right) + \left(\sin^2\theta / h^2\right), \tag{3}$$

$$2g_{12} = 2\sin\theta\cos\theta\left[\left(1/c^2\right) - \left(1/h^2\right)\right], \tag{4}$$

$$g_{22} = \left(\sin^2\theta / c^2\right) + \left(\cos^2\theta / h^2\right), \tag{5}$$

$$g_{33} = 1/v^2. \tag{6}$$

To illustrate how these equations are used, let us apply them to the olive green. The olive green standard has the CIELAB coordinates $L^*_0 = 31.71$, $a^*_0 = 3.76$, $b^*_0 = 9.31$ (see Table 9). We have $\theta = \tan^{-1}(9.31/-3.76) = 68.01^\circ$.

This gives $g_{11} = [\cos^2(-68.01)/1.13^2] + [\sin^2(-68.01)/0.60^2] = 2.50$.

Similarly, $2g_{12} = 1.39$; $g_{22} = 1.06$, $g_{33} = 0.354$. We therefore have

$$\Delta A = \left[2.50\left(a^* + 3.76\right)^2 + 1.39\left(a^* + 3.76\right)\left(b^* - 9.31\right) + 1.06\left(b^* - 9.31\right)^2 + 0.354\left(L^* - 31.71\right)^2\right]^{\frac{1}{2}}, \tag{7}$$

where a*, b* and L* are the CIELAB coordinates of the sample being judged. This equation will have to be modified each time the standard is changed. The values for c, h and v, of course, were taken from Table 8 and would remain constant irrespective of the standard.

The equations for the other colors are derived in the same way. The CIELAB coordinates for the standards are given in Table 9. We have data for two different sets of limit samples that were used with the tan; each set had its own standard. Following are the equations to be used with each of the tan standards and the equation for the dark blue.

For the standard of tan set no. 1:

$$\Delta A = \left[5.09\left(a^* - 2.05\right)^2 - 1.09\left(a^* - 2.05\right)\left(b^* - 15.64\right) + 1.01\left(b^* - 15.64\right)^2 + 0.439\left(L^* - 57.48\right)^2\right]^{\frac{1}{2}}, \tag{8}$$

For the standard of tan set no. 2:

$$\Delta A = \left[5.06\left(a^* - 2.47\right)^2 - 1.35\left(a^* - 2.47\right)\left(b^* - 15.09\right) + 1.05\left(b^* - 15.09\right)^2 + 0.439\left(L^* - 56.43\right)^2\right]^{\frac{1}{2}}, \tag{9}$$

For the dark blue:

$$\Delta A = \left[2.30\left(a^* - 0.205\right)^2 + 0.041\left(a^* - 0.205\right)\left(b^* + 11.12\right) + 1.18\left(b^* + 11.12\right)^2 + 0.943\left(L^* - 22.98\right)^2\right]^{\frac{1}{2}}, \tag{10}$$

We tried these equations on the limit samples for each color. Results are shown in Table 9. We see that all of the limit samples for the olive green and most of those for tan set no. 1 would fail the acceptability test as given. The average ΔA values are 1.65 for the olive green and 1.63 for tan set no. 1. For tan set no. 2 the average ΔA drops to 1.21, whereas for the dark blue the average ΔA is 0.76, with all but one of the limit samples passing the test.

It is widely believed in industry that for commercial customers the range of limit samples is influenced by the state of the marketplace at the time of choice. If goods are needed urgently, the limit sample might be chosen so as to reflect wider tolerances; if there is enough material and the concern is more with high quality, the reverse might be true. This is rarely true of military procurement; other factors are far more important. Industrial experience in matching a standard may result in tightening the range, as may be seen by comparing tan set no. 1 with tan set no. 2; the tolerance is considerably tighter in the latter set. It is tighter still in the dark blue set because the end use of the fabric is more critical.

Table 9 Application of Tolerance Equations to Limit Samples

DESIG.	L^*	a^*	b^*	ΔA
Olive Green				
STD	31.71	-3.76	9.31	
FY	32.29	-5.29	9.76	2.29
FS	31.72	-4.92	9.45	1.78
FG	31.87	-3.85	8.03	1.39
FB	32.35	-4.28	8.51	1.44
FR	32.10	-4.65	9.57	1.34
TG	32.23	-3.74	8.07	1.30
TY	32.43	-4.26	8.78	1.21
TB	33.79	-4.87	8.80	2.38
TS	32.65	-4.89	9.75	1.74
			Av. ± St. D.	1.65 ± 0.43
Tan, Set No. 1				
STD	57.48	2.05	15.64	
FS	56.47	3.31	16.38	2.84
FY	57.26	2.92	16.91	2.07
FG	57.54	2.49	16.36	1.08
FR	58.52	3.41	15.91	3.09
FB	58.00	2.35	15.27	0.91
TS	58.96	2.21	15.34	1.11
TY	58.50	2.53	16.74	1.51
TR	58.42	2.67	16.30	1.53
TB	57.94	2.14	15.27	0.56
			Av. ± St. D.	1.63 ± 0.87
Tan, Set. No. 2				
STD	56.43	2.47	15.09	
FS	56.06	2.80	15.07	0.79
FR	56.89	3.21	15.23	1.66
TY	56.92	1.86	15.60	1.64
FY	55.66	2.24	15.60	0.98
FG	55.97	1.82	15.12	1.50
TG	56.34	1.93	15.21	1.26
TB	57.23	2.57	14.71	0.73
FB	56.84	2.82	14.50	1.16
TS	57.83	2.56	14.63	1.09
TR	57.67	2.88	15.72	1.26
			Av. ± St. D.	1.21 ± 0.33

Table 9 (cont'd.)

DESIG.	L^*	a^*	b^*	ΔA
Dark Blue				
STD 1 *	22.90	0.40	-11.49	
STD 2 *	23.06	0.01	-10.75	
FG 1	20.93	0.16	-10.81	2.02
FG 2	22.18	-0.06	-10.79	0.94
TG	23.09	0.05	-11.48	0.47
TS	22.53	0.23	-10.89	0.51
TB	23.56	-0.03	-11.16	0.67
FB	22.70	-0.25	-11.25	0.76
FR	22.34	-0.16	-10.92	0.86
FS	22.59	0.04	-10.96	0.48
TR	22.85	0.13	-11.12	0.17
			Av. ± St. D.	0.76 ± 0.53

* Two readings on same standard; values were averaged.

The tolerances for the calculation method presented here can be varied by merely changing the limit for ΔA at which we are willing to accept samples upward or downward from 1.0. For example, if the ΔA limit is set at 1.6 instead of 1.0, we would have approximately the tolerances that obtain for the olive green limit samples. If, on the other hand, we set the ΔA limit at 0.8, we would have the more stringent tolerances that hold for the dark blue limit samples. A tolerance limit of 1.0 represents average practice, as determined by the judgments of unbiased inspectors obtained as described in the sections above. We should remember that any change in tolerance limits as just described will still maintain the relative importance of the hue, chroma and lightness directions; we are merely changing the overall tolerance.

V. CONCLUSIONS AND RECOMMENDATIONS

The methods described in this report for selecting samples for inspection, for presenting the samples to the observers, and for scoring the results and setting up the acceptability equations appear to be reliable. For the present, we recommend that every color for which tolerances are desired by the Army be handled in the same way as we have illustrated for the olive green, the tan and the dark blue. The procedure allows for flexibility in setting up the tolerances by simply changing the ΔA acceptability limit upward or downward from 1.0. This choice can be made anew each time any change in procurement requirements occurs.

It may eventually happen that enough history and experience will accumulate to make it unnecessary to obtain observers' judgments in setting up equations for each new color. We may see enough regularity in the ellipsoids to make it unnecessary to determine many more. Considerably more data like those in Table 8 would be needed, however, before we could make any such generalizations.

References

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2. D. L. MacAdam, in Color Metrics, edited by J. J. Vos, L.F.C. Friele and P. L. Walraven (AIC/Holland, Soesterberg, 1972), pp. 160-170.

3. "The Hue Angle Sort Program," pamphlet issued by Hunter Associates Laboratory, Inc.

4. J. Berkson, "Application of the Logistic Function to Bio-Assay," Am. Stat. Assn. J. 39, 257-365 (1944).

5. J. Berkson, "A Statistically Precise and Relatively Simple Method of Estimating the Bio-Assay with Quantal Response, Based on the Logistic Function," Am. Stat. Assn. J. 48, 565-599 (1953).

Appendix A. Computer program for study of relative positions of standard and limit samples (Program RAMSLEY)

This program constructs all possible tetrahedra from the nine points in color space corresponding to the standard and the eight limit samples (seven limit samples in one case). There are $\binom{9}{4} = 126$ such tetrahedra that can be constructed from the nine points.

The program takes each of the nine points in turn, and determines whether it falls inside any one of the $\binom{8}{4} = 70$ tetrahedra that are not constructed from the point in question. If the point falls inside at least one of the tetrahedra, it is obviously on the inside of the geometric figure determined by the nine points; if not, it is on the boundary. (We define the "geometric figure determined by the nine points" as the union of all 126 tetrahedra.)

The determination of whether a point falls inside a tetrahedron is made as follows: Call the four corners of the tetrahedron A, B, C and D; the point in question is called P. Choose arbitrarily the D corner of the tetrahedron and construct vectors to the other three corners (see Figure 5). Also construct a vector from point D to the point in question (dotted line). Call the vectors to the three corners $\overrightarrow{DA}$, $\overrightarrow{DB}$ and $\overrightarrow{DC}$; the vector to the point in question is $\overrightarrow{DP}$. Now the point is inside the tetrahedron if

$$p\overrightarrow{DA} + q\overrightarrow{DB} + r\overrightarrow{DC} = \overrightarrow{DP}; \quad \begin{aligned} &p > 0; \\ &q > 0; \\ &r > 0; \\ &1-p-q-r > 0. \end{aligned}$$

Expressing the vector equation in algebraic notation, if L^*_A indicates the L^* value of point A, etc., we have the following matrix equation:

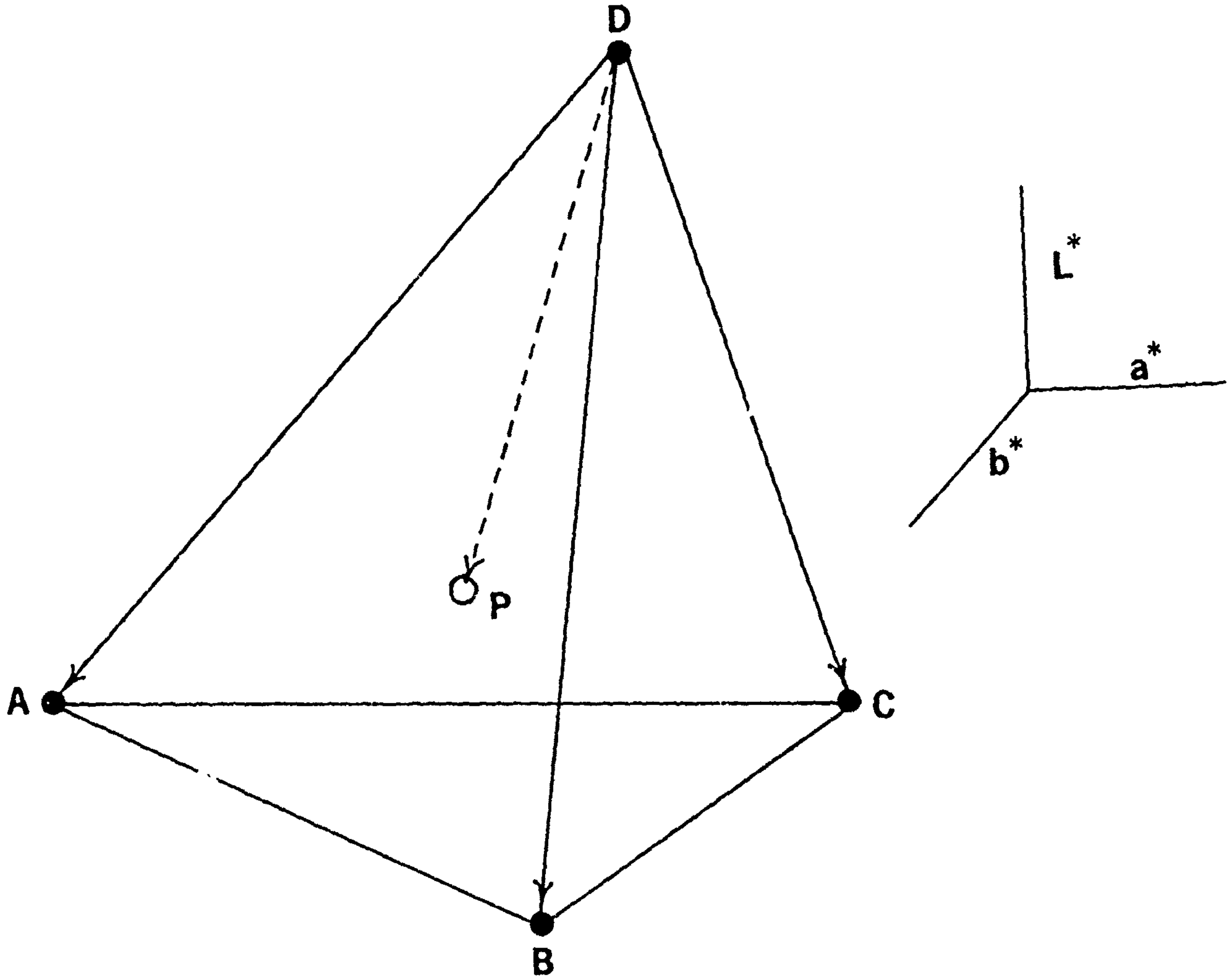

Figure 5. Vectorial representation of tetrahedron created by four samples in CIELAB Color space.

$$
\begin{bmatrix} L_A^* - L_D^* & L_B^* - L_D^* & L_C^* - L_D^* \\ a_A^* - a_D^* & a_B^* - a_D^* & a_C^* - a_D^* \\ b_A^* - b_D^* & b_B^* - b_D^* & b_C^* - b_D^* \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} L_P^* - L_D^* \\ a_P^* - a_D^* \\ b_P^* - b_D^* \end{bmatrix}
$$

The computer program forms all 126 matrices shown on the left above, and inverts them. It then chooses the 70 inverted matrices to be used for each point, multiplies them in turn into the appropriate right-hand side vector, and calculates p, q and r. If the inequalities shown above for p, q and r are satisfied in at least one of the 70 cases, the point in question is inside the geometric figure formed by the nine points.

The computer program follows.

```
      PROGRAM RAMSLEY (INPUT, OUTPUT, TAPE1, TAPE2)
C
C          FOR A SERIES OF 12 COLORS, READS THE
C          SPECTROPHOTOMETRIC CURVES OF THE STANDARDS
C          AND THE LIMIT STANDARDS, AND FOR EACH OF
C          THESE DETERMINES WHETHER ITS POINT IN
C          CIELAB COLOR SPACE LIES ON THE BOUNDARY OF
C          THE THREE-DIMENSIONAL FIGURE DETERMINED
C          BY ALL THE POINTS.
C
      DIMENSION NAME(6, 12), LIMNUM(9, 12), A(9), B(9),
     +ID(4, 126), RHS(3), C(4), LABEL(9), WORK(3), LIM(2, 9)
C
      REAL L(9), MAT(3, 3), MINV(3, 3), MINVS(3, 3, 126)
C
      DATA  NAME/8HOLIVE DR,8HAB 7, UN,8HTREATED ,8HCOTTON D,
     +8HUCK, 8.2,8H5 OZ.   ,8HTAN 46, ,8HCOTTON P,8HOPLIN, 4,
     +8H OZ.    ,8H        ,8H        ,8HTAN M-1,,8H CL. POL,
     +8HY/WOOL T,8HROP. 9 O,8HZ.      ,8H        ,8HARMY GRE,
     +8HEN 44, W,8HL GAB., ,8H13OZ.   ,8H        ,8H        ,
     +8HOLIVE GR,8HEEN 107,,8H NYLON/C,8HTN (50/5,8H0) POPLI,
     +8HN, 5 OZ.,8HBLUE 150,8HTROP. WL,8H. 10 OZ.,8H        ,
     +8H        ,8H        ,8HARMY GRE,8HEN 344, ,8HPOLYESTE,
     +8HR/WOOL G,8HABARDINE,8H 9 OZ.  ,8HARMY GRE,8HEN 344, ,
     +8HPOLYESTE,8HR/WOOL T,8HROPICAL,,8H 10 OZ. ,8HARMY GRE,
     +8HEN 44, W,8HL. SERGE,8H 15 OZ. ,8H        ,8H        ,
     +8HOLIVE GR,8HEEN 108,,8H WOOL/NY,8HLON, 16 ,8HOZ. SHIR,
     +8HTING    ,8HOLIVE GR,8HEEN 107,,8H CTN. SA,8HTEEN, 8.,
     +8H2 OZ.   ,8H        ,8HBLUE 151,8H, TROP. ,8HWL., 10 ,
     +8HOZ.     ,8H        ,8H        /
C
      DATA LIMNUM/1,2,3,4,5,7,8,9,10,1,2,5,6,7,8,10,11,3,
     +1,2,3,4,5,7,8,9,10,1,2,6,4,5,7,11,9,10,1,2,3,4,5,7,8,9,10,
     +1,2,6,4,5,7,11,9,10,1,2,6,4,5,7,11,9,10,1,2,6,4,5,7,11,9,10,
     +1,6,4,5,7,2,11,9,10,1,2,3,4,5,7,8,9,10,1,2,3,5,4,7,8,9,10,
     +1,2,6,4,5,7,11,9,10/
C
C          START LARGE DO LOOP BASED ON 12 SAMPLES
C
      REWIND 1
      DO 500 NSTD = 1, 12
C
C          START DO LOOP BASED ON NINE STANDARDS AND LIMIT
C          SAMPLES (EIGHT IN THE CASE OF SAMPLE NO. 2)
C
      NP = 9
      IF (NSTD . EQ . 2) NP = 8
      DO 190 M = 1, NP
      READ (1, 1001) L(M), A(M), B(M)
C
C          ASSIGN NAMES TO STANDARDS AND LIMIT STANDARDS.
C
      INDEX = LIMNUM(M, NSTD)
      GO TO (171, 172, 173, 174, 175, 176, 177, 178, 179, 180,
     +181), INDEX
  171 LIM(1, M) = 8HSTANDARD
      LIM(2, M) = 8H
      GO TO 190
  172 LIM(1, M) = 8HTHIN STA
      LIM(2, M) = 8HNDARD
      GO TO 190
  173 LIM(1, M) = 8HTHIN YEL
      LIM(2, M) = 8HLOW
```

```
      GO TO 190
  174 LIM(1, M) = 8HTHIN GRE
      LIM(2, M) = 8HEN
      GO TO 190
  175 LIM(1, M) = 8HTHIN RED
      LIM(2, M) = 8H
      GO TO 190
  176 LIM(1, M) = 8HTHIN BLU
      LIM(2, M) = 8HE
      GO TO 190
  177 LIM(1, M) = 8HFULL STA
      LIM(2, M) = 8HNDARD
      GO TO 190
  178 LIM(1, M) = 8HFULL YEL
      LIM(2, M) = 8HLOW
      GO TO 190
  179 LIM(1, M) = 8HFULL GRE
      LIM(2, M) = 8HEN
      GO TO 190
  180 LIM(1, M) = 8HFULL RED
      LIM(2, M) = 8H
      GO TO 190
  181 LIM(1, M) = 8HFULL BLU
      LIM(2, M) = 8HE
  190 CONTINUE
C
C         CREATE MATRICES REPRESENTING TETRAHEDRA FOR ALL
C         THE POSSIBLE COMBINATIONS OF THE POINTS TAKEN
C         FOUR AT A TIME
C
      NMAT = 0
      NPM1 = NP - 1
      NPM2 = NP - 2
      NPM3 = NP - 3
      DO 200 J1 = 1, NPM3
      J1P = J1 + 1
      DO 200 J2 = J1P, NPM2
      J2P = J2 + 1
      DO 200 J3 = J2P, NPM1
      J3P = J3 + 1
      DO 200 J4 = J3P, NP
      NMAT = NMAT + 1
      ID(1, NMAT) = J1
      ID(2, NMAT) = J2
      ID(3, NMAT) = J3
      ID(4, NMAT) = J4
      MAT(1, 1) = L(J1) - L(J4)
      MAT(1, 2) = L(J2) - L(J4)
      MAT(1, 3) = L(J3) - L(J4)
      MAT(2, 1) = A(J1) - A(J4)
      MAT(2, 2) = A(J2) - A(J4)
      MAT(2, 3) = A(J3) - A(J4)
      MAT(3, 1) = B(J1) - B(J4)
      MAT(3, 2) = B(J2) - B(J4)
      MAT(3, 3) = B(J3) - B(J4)
C
C         INVERT MATRICES AND STORE THEM IN MEMORY.
C
      CALL LINV1F(MAT, 3, 3, MINV, 0, WORK, IER)
      DO 200 J = 1, 3
      DO 200 K = 1, 3
  200 MINVS(J, K, NMAT) = MINV(J, K)
C
C         TRY EACH POINT SEPARATELY WITH EACH OF THE
C         INVERTED MATRICES, TO SEE IF IT FALLS WITHIN
C         THE TETRAHEDRON DEFINED BY THE FOUR POINTS THAT
```

```
C        WENT INTO THE MATRIX. IF SO, LABEL THE POINT
C        AS BEING INSIDE THE BOUNDARY.
C
      DO 300 J = 1, 9
      DO 270 NMAT = 1, 126
      IF (J . EQ . ID(1, NMAT) . OR . J . EQ . ID(2, NMAT) . OR .
     + J . EQ . ID(3, NMAT) . OR . J . EQ . ID(4, NMAT)) GO TO 270
      JJ = ID(4, NMAT)
      RHS(1) = L(J) - L(JJ)
      RHS(2) = A(J) - A(JJ)
      RHS(3) = B(J) - B(JJ)
      DO 250 K = 1, 3
      C(K) = 0.
      DO 250 KK = 1, 3
  250 C(K) = C(K) + MINVS(K, KK, NMAT) * RHS(KK)
      C(4) = 1. - C(1) - C(2) - C(3)
      IF (C(1) . GT . 0. . AND . C(2) . GT . 0. . AND .
     + C(3) . GT . 0. . AND . C(4) . GT . 0.) GO TO 280
  270 CONTINUE
      LABEL(J) = 2
      GO TO 300
  280 LABEL(J) = 1
  300 CONTINUE
C
C        PRINT RESULTS
C
      PRINT 1002, (NAME(J, NSTD), J = 1, 6)
      PRINT 1008
      DO 400 J = 1, NP
      INDEX = LABEL(J)
      GO TO (310, 320), INDEX
  310 PRINT 1003, LIM(1, J), LIM(2, J), L(J), A(J), B(J)
      GO TO 400
  320 PRINT 1004, LIM(1, J), LIM(2, J), L(J), A(J), B(J)
  400 CONTINUE
      IF (NSTD . EQ . 4 . OR . NSTD . EQ . 8 . OR .
     +NSTD . EQ . 12) PRINT 1010
      IF (NSTD . EQ . 2) PRINT 1006
  500 CONTINUE
      STOP
C
C        FORMAT STATEMENTS
C
 1001 FORMAT(3F8.2)
 1002 FORMAT(///6X, 6A8)
 1003 FORMAT(1X, 2A8, F7.2, 2F8.2, 3X, *NOT ON BOUNDARY*)
 1004 FORMAT(1X, 2A8, F7.2, 2F8.2, 3X, *ON BOUNDARY*)
 1006 FORMAT(1H )
 1008 FORMAT(/3X, *SAMPLE*, 12X, *L*, 7X, *A*, 7X, *B*,
     +7X, *POSITION*/)
 1010 FORMAT(/)
      END
```

Appendix B. Computer program for determination of minimum non-tilting ellipsoid containing standard and limit samples (Programs ELPSOID and ELPSOD2)

A non-tilting ellipsoid (in the sense described in the main section of this report) is completely determined by seven parameters: the three position coordinates of the center, the three semiaxis lengths, and the angle between the major horizontal axis and the a^* axis. For Program ELPSOID, all seven parameters are adjustable. For Program ELPSOD2, only six parameters are adjustable; the axis angle is constrained by the fact that the major horizontal axis lies along a line connecting the center of the ellipsoid with the L^* axis.

Consider a non-tilting ellipsoid located somewhere in the a^*-b^*-L^* coordinate system. Set up another coordinate system to coincide with the three axes of the ellipsoid. The equation of the ellipsoid in this second coordinate system is

$$\frac{x^2}{p^2} + \frac{y^2}{q^2} + \frac{z^2}{r^2} = 1, \qquad \text{(B-1)}$$

where x, y and z are the coordinates of any point on the ellipsoid relative to the second coordinate system, and p, q and r are the semiaxis lengths. We consider that x and y are the axes parallel to the a^*-b^* plane and z is the axis that is parallel to the L^* axis. The major horizontal semiaxis, of length 2p, lies along the x axis.

If θ is the angle going from the positive branch of the a^* axis counter-clockwise to the positive branch of the x axis, and if a_c^*, b_c^* and L_c^* are the coordinates of the center of the ellipsoid in the a^*-b^*-L^* system, then the following transformation equations apply:

$$x = (a^* - a_c^*) \cos \theta + (b^* - b_c^*) \sin \theta,$$

$$y = -(a^* - a_c^*) \sin \theta + (b^* - b_c^*) \cos \theta, \qquad \text{(B-2)}$$

$$z = L - L_c$$

Substituting these equations into the equation of the ellipsoid given above and collecting terms, we have

$$\left(\frac{\cos^2\theta}{p^2} + \frac{\sin^2\theta}{q^2}\right)\left(a^* - a_c^*\right)^2$$

$$+ 2\sin\theta\cos\theta\left(\frac{1}{p^2} - \frac{1}{q^2}\right)\left(a^* - a_c^*\right)\left(b^* - b_c^*\right) \tag{B-3}$$

$$+ \left(\frac{\sin^2\theta}{p^2} + \frac{\cos^2\theta}{q^2}\right)\left(b^* - b_c^*\right)^2$$

$$+ \frac{1}{r^2}\left(L^* - L_c^*\right)^2 = 1.$$

This is the equation for the ellipsoid in the a^*-b^*-L^* coordinate system incorporating the seven parameters mentioned above. The position parameters for the center are a_c^*, b_c^* and L_c^*; the semiaxis lengths are p, q and r; the angle between the major horizontal axis and the a^* axis is θ.

Program ELPSOID works in the following way: The se n parameters are systematically varied, one at a time. After each change in the parameter., they are inserted in Equation B-3 to give what may be called a reference ellipsoid. All the points (a^*, b^*, L^*) that satisfy Equation B-3 fall on the surface of this reference ellipsoid.

For each parameter change and associated reference ellipsoid, the program then enters a subroutine called SIZER. This subroutine goes through the nine points for the standard and the limit samples, and substitutes the coordinates of each point at a time into the equation for the reference ellipsoid. Consider one of these points. Since this point is probably not on the reference ellipsoid, the right hand side of the equation will not be unity but some value that we will call S^2. Dividing Equation B-3 through by S^2, and setting $p' = pS$, $q' = qS$ and $r' = rS$, we obtain a new equation just like Equation B-3 but with p' replacing p, q' replacing q, and r' replacing r, and with unity back on the right hand side.

This new equation defines a new ellipsoid that is concentric with the reference ellipsoid, that has the point in question on its surface and that has the same ratios of axis lengths as those of the reference ellipsoid, since the new axes are merely the old axes multiplied by S. The volume of an ellipsoid is equal to 4/3 π multiplied by the product of the three semiaxes; the new ellipsoid therefore has a volume equal to 4/3 π times the product of the p, q and r parameters times S^3.

The subroutine goes through all nine points in this way, and computes nine corresponding ellipsoids, all concentric with and similar (in the geometric sense) to the reference ellipsoid. It then returns to the main program the volume of the largest of these concentric ellipsoids, since that is the one that contains all the points. If this volume is smaller than the smallest volume previously determined for other parameter variations, the current variation was favorable; otherwise not. The systematic parameter variation continues until no parameter change can produce a smaller ellipsoid returned by the subroutine. The program then normalizes the current parameters by multiplying the p, q and r values by the S value for the ellipsoid that was chosen, and finally prints out the ellipsoid characteristics.

In Program ELPSOD2, the angle θ is always fixed by

$$\theta = \tan^{-1} (b_c^*/a_c^*) \; . \tag{B-4}$$

It is therefore obviously a hue angle. Each time the a_c^* or b_c^* coordinate is varied, the angle θ must be recalculated. Otherwise, the program runs like Program ELPSOID.

It is convenient to define the coefficients in Equation B-3 as follows:

$$g_{11} = (\cos^2 \theta/p^2) + (\sin^2 \theta/q^2), \tag{B-5}$$

$$2g_{12} = 2 \sin \theta \cos \theta \left[(1/p^2) - (1/q^2)\right] \; , \tag{B-6}$$

$$g_{22} = (\sin^2 \theta/p^2 + (\cos^2 \theta \; q^2), \tag{B-7}$$

$$g_{33} = 1/r^2 \; . \tag{B-8}$$

With these substitutions, Equation B-3 omes

$$g_{11}\,(a^* - a_c^*)^2 + 2\,g_{12}\,(a^* - a_c^*)\,(b^* - b_c^*) + g_{22}\,(b^* - b_c^*)^2 + g_{33}\,(L^* - L_c^*)^2 = 1. \qquad \text{(B-9)}$$

This is the terminology that is used in the computer programs.

```
      PROGRAM ELPSOID (INPUT, OUTPUT, TAPE3)
C
C         FOR A SERIES OF POINTS IN CIELAB COLOR SPACE,
C         DETERMINES THE SMALLEST NON-TILTING ELLIPSOID
C         THAT CONTAINS THE POINTS. THE ELLIPSOID MAY
C         BE INCLINED WITH RESPECT TO THE HORIZONTAL AXIS,
C         BUT MAY NOT TILT EITHER FORWARD OR SIDEWAYS.
C
      DIMENSION PAR(7), HOWFAR(9), DELTA(7), DIV(4), NAME(6, 12),
     +SIGN(2)
C
      COMMON CIE(3, 9), NP
C
      LOGICAL CHANGE, ACTION
C
      DATA DELTA/0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.017453293/,
     +DIV/1., 2., 5., 10./, SIGN/1., -1./
C
      DATA  NAME/8HOLIVE DR,8HAB 7, UN,8HTREATED ,8HCOTTON D,
     +8HUCK, 8.2,8H5 OZ.   ,8HTAN 46, ,8HCOTTON P,8HOPLIN, 4,
     +8H OZ.    ,8H        ,8H        ,8HTAN M-1,,8H CL. POL,
     +8HY/WOOL T,8HROP. 9 O,8HZ.      ,8H        ,8HARMY GRE,
     +8HEN 44, W,8HL GAB., ,8H13OZ.   ,8H        ,8H        ,
     +8HOLIVE GR,8HEEN 107,,8H NYLON/C,8HTN (50/5,8H0) POPLI,
     +8HN, 5 OZ.,8HBLUE 150,8HTROP. WL,8H. 10 OZ.,8H        ,
     +8H        ,8H        ,8HARMY GRE,8HEN 344, ,8HPOLYESTE,
     +8HR/WOOL G,8HABARDINE,8H 9 OZ.  ,8HARMY GRE,8HEN 344, ,
     +8HPOLYESTE,8HR/WOOL T,8HROPICAL,,8H 10 OZ. ,8HARMY GRE,
     +8HEN 44, W,8HL. SERGE,8H 15 OZ. ,8H        ,8H        ,
     +8HOLIVE GR,8HEEN 108,,8H WOOL/NY,8HLON, 16 ,8HOZ. SHIR,
     +8HTING    ,8HOLIVE GR,8HEEN 107,,8H CTN. SA,8HTEEN, 8.,
     +8H2 OZ.   ,8H        ,8HBLUE 151,8H, TROP. ,8HWL., 10 ,
     +8HOZ.     ,8H        ,8H        /
C
C         READ LAB VALUES OF STANDARDS (BUT IN THE ORDER ABL)
C
      REWIND 3
      DO 600 NSTD = 1, 12
      IF (NSTD . NE . 2) NP = 9
      IF (NSTD . EQ . 2) NP = 8
      READ (3, 1001)(CIE(3, J), CIE(1, J), CIE(2, J), J = 1, NP)
C
C         ASSIGN STARTING VALUES TO THE SEVEN PARAMETERS
C
      HDMAX = 0.
      VDMAX = 0.
      NPM1 = NP - 1
      DO 150 J = 1, NPM1
      JP1 = J + 1
      DO 150 K = JP1, NP
      HD = SQRT((CIE(1, J) - CIE(1, K)) ** 2 + (CIE(2, J)
     + - CIE(2, K)) ** 2)
      IF (HD . LE . HDMAX) GO TO 130
      HDMAX = HD
      JHMAX = J
      KHMAX = K
  130 VD = ABS(CIE(3, J) - CIE(3, K))
      IF (VD . LE . VDMAX) GO TO 150
      VDMAX = VD
      JVMAX = J
      KVMAX = K
  150 CONTINUE
      PAR(1) = HDMAX / 2.
```

```
C      HORIZONTAL MAJOR SEMIAXIS
      PAR(2) = PAR(1) / 2.
C      HORIZONTAL MINOR SEMIAXIS
      PAR(3) = JDMAX / 2.
C      VERTICAL SEMIAXIS
      PAR(4) = 0.5 * (CIE(1, JHMAX) + CIE(1, KHMAX))
C      A STAR COORDINATE OF CENTER OF ELLIPSOID
      PAR(5) = 0.5 * (CIE(2, JHMAX) + CIE(2, KHMAX))
C      B STAR COORDINATE OF CENTER OF ELLIPSOID
      PAR(6) = 0.5 * (CIE(3, JVMAX) + CIE(3, KVMAX))
C      L STAR COORDINATE OF CENTER OF ELLIPSOID
      PAR(7) = ATAN((CIE(2, JHMAX) - CIE(2, KHMAX)) /
     + (CIE(1, JHMAX) - CIE(1, KHMAX)))
C      ANGLE OF HORIZONTAL MAJOR AXIS TO A STAR AXIS
C
C         DO LOOP FOR IDIOT SEARCH PROCEDURE. CHANGE ONE
C         PARAMETER AT A TIME AND SEE IF THE ELLIPSOID
C         GETS SMALLER. IF SO, START OVER. EVENTUALLY
C         MAKE THE PARAMETER CHANGE INCREMENTS SMALLER
C         AND SMALLER.
C
      CALL SIZER (PAR, ELLMIN)
      DO 450 NDIV = 1, 4
  160 CHANGE = .FALSE.
      DO 400 NPAR = 1, 7
      DEL = DELTA(NPAR) / DIV(NDIV)
      DO 400 NSIGN = 1, 2
      IF (NSIGN . EQ . 1) GO TO 170
      IF (ACTION) GO TO 400
  170 ACTION = .FALSE.
  180 PAR(NPAR) = PAR(NPAR) + DEL * SIGN(NSIGN)
      IF (PAR(1) . LT . 0. . OR . PAR(2) . LT . 0.
     +. OR . PAR(3) . LT . 0.) GO TO 390
      CALL SIZER (PAR, ELLIE)
      IF (ELLIE . GT . ELLMIN) GO TO 390
      ELLMIN = ELLIE
      CHANGE = .TRUE.
      ACTION = .TRUE.
      GO TO 180
  390 PAR(NPAR) = PAR(NPAR) - DEL * SIGN(NSIGN)
  400 CONTINUE
      IF (CHANGE) GO TO 160
  450 CONTINUE
C
C         NORMALIZE ELLIPSOID SEMIAXES BEFORE PRINTING OUT
C         INFORMATION. SEMIAXES ARE ADJUSTED SO AS TO MAKE
C         RIGHT HAND SIDE OF ELLIPSOID EQUATION UNITY.
C
      CALL SIZER (PAR, ELLIE)
      ETEMP = PAR(1) * PAR(2) * PAR(3)
      SCFACTR = (ELLIE / ETEMP) ** (1. / 3.)
      DO 500 J = 1, 3
  500 PAR(J) = PAR(J) * SCFACTR
      IF (PAR(1) . GT . PAR(2)) GO TO 510
      TEMP = PAR(2)
      PAR(2) = PAR(1)
      PAR(1) = TEMP
      PAR(7) = PAR(7) - 0.5 * 3.141592654
C
C         CALCULATE ELLIPSOID PARAMETERS FOR OUTPUT
C
  510 ANG = PAR(7) * 180. / 3.141592654
      IF (ANG . GT . 90) ANG = ANG - 180.
      SINE = SIN(PAR(7))
      COSINE = COS(PAR(7))
      G11 = (COSINE / PAR(1)) ** 2 + (SINE / PAR(2)) ** 2
```

```
      TWOG12 = 2. * SINE * COSINE * (1. / PAR(1) ** 2 -
     + 1. / PAR(2) ** 2)
      G22 = (SINE / PAR(1)) ** 2 + (COSINE / PAR(2)) ** 2
      G33 = 1. / PAR(3) ** 2
      VOL = (4. / 3.) * 3.141592654 * PAR(1) * PAR(2) * PAR(3)
C
C         PRINT OUT RESULTS
C
      PRINT 1002, (NAME(J, NSTD), J = 1, 6)
      PRINT 1003, PAR(6), PAR(4), PAR(5)
      PRINT 1004, G11, TWOG12, G22, G33
      PRINT 1005, PAR(1), ANG, PAR(2), PAR(3)
      PRINT 1006, VOL
      DO 530 J = 1, NP
      DEL1 = CIE(1, J) - PAR(4)
      DEL2 = CIE(2, J) - PAR(5)
      DEL3 = CIE(3, J) - PAR(6)
  530 HOWFAR(J) = G11 * DEL1 ** 2 + TWOG12 * DEL1 * DEL2 +
     + G22 * DEL2 ** 2 + G33 * DEL3 ** 2
      PRINT 1007, (CIE(1, J), CIE(2, J), CIE(3, J), HOWFAR(J),
     + J = 1, NP)
  600 CONTINUE
      PRINT 1008
C
C         FORMAT STATEMENTS
C
 1001 FORMAT(3F8.2)
 1002 FORMAT(//////* ACCEPTABILITY ELLIPSOID*/1X,6A8)
 1003 FORMAT(//24H CENTER OF ELLIPSOID: L*, F18.2/22X,
     + 2HA*, F18.2/22X, 2HB*, F18.2)
 1004 FORMAT(/* COEFFICIENTS:*,8X, *G11*, E23.4/21X,
     + *2G12*, E23.4/22X, *G22*, E23.4/22X, *G33*, E23.4)
 1005 FORMAT(/* SEMIMAJOR HORIZONTAL AXIS, LENGTH*, F8.2/
     + * SEMIMAJOR HORIZONTAL AXIS, ANGLE*, F8.1/* SEMIMINOR*,
     + * HORIZONTAL AXIS, LENGTH*, F8.2/* SEMIVERTICAL AXIS,*
     + * LENGTH*, F16.2)
 1006 FORMAT(/* VOLUME OF ELLIPSOID*,F22.2)
 1007 FORMAT(/* DISTANCE OF POINT FROM CENTER OF ELLIPSOID*
     + * IN ELL UNITS*//10X, *COORDINATES OF POINT    DISTANCE*,
     + //11X,2HA*,6X,2HB*,6X,2HL*/(6X, 3F8.2, F11.5))
 1008 FORMAT(/////////)
      END
C
C
      SUBROUTINE SIZER (PAR, ELLIE)
C
C         DETERMINES THE SIZE OF THE ELLIPSOID PASSING THROUGH
C         EACH OF THE STANDARDS WITH THE CURRENT PARAMETERS, AND
C         SELECTS THE ELLIPSOID OF LARGEST SIZE AMONG THEM.
C
      DIMENSION PAR(7)
C
      COMMON CIE(3, 9), NP
C
      SINE = SIN(PAR(7))
      COSINE = COS(PAR(7))
      G11 = (COSINE / PAR(1)) ** 2 + (SINE / PAR(2)) ** 2
      TWOG12 = 2. * SINE * COSINE * (1. / PAR(1) ** 2 -
     + 1. / PAR(2) ** 2)
      G22 = (SINE / PAR(1)) ** 2 + (COSINE / PAR(2)) ** 2
      G33 = 1. / PAR(3) ** 2
      RHSMAX = 0.
      DO 150 J = 1, NP
      DELTA = CIE(1, J) - PAR(4)
      DELTB = CIE(2, J) - PAR(5)
      DELTL = CIE(3, J) - PAR(6)
```

```
      RHS = G11 * DELTA ** 2 + TWOG12 * DELTA * DELTB +
     + G22 * DELTB ** 2 + G33 * DELTL ** 2
      IF (RHS . LT . RHSMAX) GO TO 150
      RHSMAX = RHS
  150 CONTINUE
      ELLIE = RHSMAX ** 1.5 * PAR(1) * PAR(2) * PAR(3)
      RETURN
      END
```

```
      PROGRAM ELPSOD2 (INPUT, OUTPUT, TAPE3)
C
C          FOR A SERIES OF POINTS IN CIELAB COLOR SPACE,
C          DETERMINES THE SMALLEST NON-TILTING ELLIPSOID
C          THAT CONTAINS THE POINTS. THE ELLIPSOID MAY
C          NOT TILT EITHER FORWARD OR SIDEWAYS, AND MUST
C          HAVE ITS MAJOR AXIS INCLINED IN THE DIRECTION
C          OF THE HUE ANGLE OF THE STANDARD.
C
      DIMENSION PAR(7), HOWFAR(9), DIV(4), NAME(6, 12), SIGN(2)
C
      COMMON CIE(3, 9), NP
C
      LOGICAL CHANGE, ACTION
C
      DATA DELTA/0.1/, DIV/1., 2., 5., 10./, SIGN/1., -1./
C
      DATA  NAME/8HOLIVE DR,8HAB 7, UN,8HTREATED ,8HCOTTON D,
     +8HUCK, 8.2,8H5 OZ.   ,8HTAN 46, ,8HCOTTON P,8HOPLIN, 4,
     +8H OZ.    ,8H        ,8HTAN M-1,,8H CL. POL,
     +8HY/WOOL T,8HROP. 9 O,8HZ.      ,8H        ,8HARMY GRE,
     +8HEN 44, W,8HL GAB., ,8H1?OZ.   ,8H        ,8H        ,
     +8HOLIVE GR,8HEEN 107,,8H NYLON/C,8HTN (50/5,8H0) POPLI,
     +8HN, 5 OZ.,8HBLUE 150,8HTROP. WL,8H. 10 OZ.,8H        ,
     +8H        ,8H        ,8HARMY GRE,8HEN 344, ,8HPOLYESTE,
     +8HR/WOOL G,8HABARDINE,8H 9 OZ.  ,8HARMY GRE,8HEN 344, ,
     +8HPOLYESTE,8HR/WOOL T,8HROPICAL,,8H 10 OZ. ,8HARMY GRE,
     +8HEN 44, W,8HL. SERGE,8H 15 OZ. ,8H        ,8H        ,
     +8HOLIVE GR,8HEEN 108,,8H WOOL/NY,8HLON, 16 ,8HOZ. SHIR,
     +8HTING    ,8HOLIVE GR,8HEEN 107,,8H CTN. SA,8HTEEN, 8.,
     +8H2 OZ.   ,8H        ,8HBLUE 151,8H, TROP. ,8HWL., 10 ,
     +8HOZ.     ,8H        ,8H        /
C
C          READ LAB VALUES OF STANDARDS (BUT IN THE ORDER ABL)
C
      REWIND 3
      DO 600 NSTD = 1, 12
      IF (NSTD . NE . 2) NP = 9
      IF (NSTD . EQ . 2) NP = 8
      READ (3, 1001)(CIE(3, J), CIE(1, J), CIE(2, J), J = 1, NP)
C
C          ASSIGN STARTING VALUES TO THE SEVEN PARAMETERS
C
      PAR(4) = 0.
      PAR(5) = 0.
      PAR(6) = 0.
      DO 120 J = 1, NP
      PAR(4) = PAR(4) + CIE(1, J) / NP
C      A STAR COORDINATE OF CENTER OF ELLIPSOID
      PAR(5) = PAR(5) + CIE(2, J) / NP
C      B STAR COORDINATE OF CENTER OF ELLPISOID
      PAR(6) = PAR(6) + CIE(3, J) / NP
C      L STAR COORDINATE OF CENTER OF ELLIPSOID
  120 CONTINUE
      D1MAX = 0.
      D2MAX = 0.
      D3MAX = 0.
      DO 150 J = 1, NP
      D = SQRT((PAR(4) - CIE(1, J)) ** 2 + (PAR(5)
     + - CIE(2, J)) ** 2)
      ALPHA = ATAN((PAR(5) - CIE(2, J)) / (PAR(4)
     + - CIE(1, J)))
      THETA = ATAN(PAR(5) / PAR(4))
```

```
      PHI = ALPHA - THETA
      D1 = ABS(D * COS(PHI))
      IF (D1 . LE . D1MAX) GO TO 130
      D1MAX = D1
  130 D2 = ABS(D * SIN(PHI))
      IF (D2 . LE . D2MAX) GO TO 140
      D2MAX = D2
  140 D3 = ABS(PAR(6) - CIE(3, J))
      IF (D3 . LE . D3MAX) GO TO 150
      D3MAX = D3
  150 CONTINUE
      PAR(1) = D1MAX
C       HORIZONTAL MAJOR SEMIAXIS
      PAR(2) = D2MAX
C       HORIZONTAL MINOR SEMIAXIS
      PAR(3) = D3MAX
C       VERTICAL SEMIAXIS
      PAR(7) = THETA
C       ANGLE OF HORIZONTAL MAJAR AXIS TO A STAR AXIS
C
C           DO LOOP FOR IDIOT SEARCH PROCEDURE. CHANGE ONE
C           PARAMETER AT A TIME AND SEE IF THE ELLIPSOID
C           GETS SMALLER. IF SO, START OVER. EVENTUALLY
C           MAKE THE PARAMETER CHANGE INCREMENTS SMALLER
C           AND SMALLER.
C
      CALL SIZER (PAR, ELLMIN)
      DO 450 NDIV = 1, 4
  160 CHANGE = .FALSE.
      DO 400 NPAR = 1, 6
      DEL = DELTA / DIV(NDIV)
      DO 400 NSIGN = 1, 2
      IF (NSIGN . EQ . 1) GO TO 170
      IF (ACTION) GO TO 400
  170 ACTION = .FALSE.
  180 PAR(NPAR) = PAR(NPAR) + DEL * SIGN(NSIGN)
      IF (PAR(1) . LT . 0. . OR . PAR(2) . LT . 0.
     +. OR . PAR(3) . LT . 0.) GO TO 390
      IF (NPAR . EQ . 4 . OR . NPAR . EQ . 5)
     + PAR(7) = ATAN(PAR(5) / PAR(4))
      CALL SIZER (PAR, ELLIE)
      IF (ELLIE . GT . ELLMIN) GO TO 390
      ELLMIN = ELLIE
      CHANGE = .TRUE.
      ACTION = .TRUE.
      GO TO 180
  390 PAR(NPAR) = PAR(NPAR) - DEL * SIGN(NSIGN)
      IF (NPAR . EQ . 4 . OR . NPAR . EQ . 5)
     + PAR(7) = ATAN(PAR(5) / PAR(4))
  400 CONTINUE
      IF (CHANGE) GO TO 160
  450 CONTINUE
C
C           NORMALIZE ELLIPSOID SEMIAXES BEFORE PRINTING OUT
C           INFORMATION. SEMIAXES ARE ADJUSTED SO AS TO MAKE
C           RIGHT HAND SIDE OF ELLIPSOID EQUATION UNITY.
C
      CALL SIZER (PAR, ELLIE)
      ETEMP = PAR(1) * PAR(2) * PAR(3)
      SCFACTR = (ELLIE / ETEMP) ** (1. / 3.)
      DO 500 J = 1, 3
  500 PAR(J) = PAR(J) * SCFACTR
C
C           CALCULATE ELLIPSOID PARAMETERS FOR OUTPUT
C
      ANG = PAR(7) * 180. / 3.141592654
```

```
      IF (ANG . GT . 90) ANG = ANG - 180.
      SINE = SIN(PAR(7))
      COSINE = COS(PAR(7))
      G11 = (COSINE / PAR(1)) ** 2 + (SINE / PAR(2)) ** 2
      TWOG12 = 2. * SINE * COSINE * (1. / PAR(1) ** 2 -
     + 1. / PAR(2) ** 2)
      G22 = (SINE / PAR(1)) ** 2 + (COSINE / PAR(2)) ** 2
      G33 = 1. / PAR(3) ** 2
      RATIO1 = PAR(3) / PAR(2)
      RATIO2 = PAR(1) / PAR(2)
      VOL = (4. / 3.) * 3.141592654 * PAR(1) *PAR(2) * PAR(3)
C
C         PRINT OUT RESULTS
C
      PRINT 1002, (NAME(J, NSTD), J = 1, 6)
      PRINT 1003, PAR(6), PAR(4), PAR(5)
      PRINT 1004, G11, TWOG12, G22, G33
      PRINT 1005, PAR(1), ANG, PAR(2), PAR(3)
      PRINT 1006, RATIO1, RATIO2
      PRINT 1007, VOL
      DO 530 J = 1, NP
      DEL1 = CIE(1, J) - PAR(4)
      DEL2 = CIE(2, J) - PAR(5)
      DEL3 = CIE(3, J) - PAR(6)
  530 HOWFAR(J) = G11 * DEL1 ** 2 + TWOG12 * DEL1 * DEL2 +
     + G22 * DEL2 ** 2 + G33 * DEL3 ** 2
      PRINT 1008, (CIE(1, J), CIE(2, J), CIE(3, J), HOWFAR(J),
     + J = 1, NP)
  600 CONTINUE
      PRINT 1009
C
C         FORMAT STATEMENTS
C
 1001 FORMAT(3F8.2)
 1002 FORMAT(//////* ACCEPTABILITY ELLIPSOID*/1X,6A8)
 1003 FORMAT(//24H CENTER OF ELLIPSOID: L*, F18.2/22X,
     + 2HA*, F18.2/22X, 2HB*, F18.2)
 1004 FORMAT(/* COEFFICIENTS:*,8X, *G11*, E23.4/21X,
     + *2G12*, E23.4/22X, *G22*, E23.4/22X, *G33*, E23.4)
 1005 FORMAT(/* SEMIMAJOR HORIZONTAL AXIS, LENGTH*, F8.2/
     + * SEMIMAJOR HORIZONTAL AXIS, ANGLE*, F8.1/* SEMIMINOR*,
     + * HORIZONTAL AXIS, LENGTH*, F8.2/* SEMIVERTICAL AXIS,*
     + * LENGTH*, F16.2)
 1006 FORMAT(/* LIGHTNESS : CHROMA : HUE RATIOS*,
     + F10.2, * :*, F5.2, * : 1.00*)
 1007 FORMAT(/* VOLUME OF ELLPISOID*, F22.2)
 1008 FORMAT(/* DISTANCE OF POINT FROM CENTER OF ELLIPSOID*
     + * IN ELL UNITS*//10X, *COORDINATES OF POINT    DISTANCE*,
     + //11X,2HA*,6X,2HB*,6X,2HL*/(6X, 3F8.2, F11.5))
 1009 FORMAT(////////)
      END
C
C
      SUBROUTINE SIZER (PAR, ELLIE)
C
C         DETERMINES THE SIZE OF THE ELLIPSOID PASSING THROUGH
C         EACH OF THE STANDARDS WITH THE CURRENT PARAMETERS, AND
C         SELECTS THE ELLIPSOID OF LARGEST SIZE AMONG THEM.
C
      DIMENSION PAR(7)
C
      COMMON CIE(3, 9), NP
C
      SINE = SIN(PAR(7))
      COSINE = COS(PAR(7))
      G11 = (COSINE / PAR(1)) ** 2 + (SINE / PAR(2)) ** 2
```

```
      TWOG12 = 2. * SINE * COSINE * (1. / PAR(1) ** 2 -
     + 1. / PAR(2) ** 2)
      G22 = (SINE / PAR(1)) ** 2 + (COSINE / PAR(2)) ** 2
      G33 = 1. / PAR(3) ** 2
      RHSMAX = 0.
      DO 150 J = 1, NP
      DELTA = CIE(1, J) - PAR(4)
      DELTB = CIE(2, J) - PAR(5)
      DELTL = CIE(3, J) - PAR(6)
      RHS = G11 * DELTA ** 2 + TWOG12 * DELTA * DELTB +
     + G22 * DELTB ** 2 + G33 * DELTL ** 2
      IF (RHS . LT . RHSMAX) GO TO 150
      RHSMAX = RHS
  150 CONTINUE
      ELLIE = RHSMAX ** 1.5 * PAR(1) * PAR(2) * PAR(3)
      RETURN
      END
```

Appendix C. Computer program for rearranging file in numerical order (Program (REARRAN)

```
      PROGRAM REARRAN (INPUT, OUTPUT, TAPE1, TAPE3)
      DIMENSION I(200), A(200), B(200)
      REAL L(200)
      LOGICAL OK
      REWIND 3
      J = 1
  100 READ (3, 1001) I1, I2, L(J), A(J), B(J)
 1001 FORMAT(I5, 1X, I3, F11.0, F15.0, F10.0)
      IF (I1 . EQ . 999) GO TO 120
      I(J) = 1000 * I1 + I2
      J = J + 1
      GO TO 100
  120 NCOLORS = J - 1
      NMIN1 = NCOLORS - 1
      ITER = 0
  130 OK = .TRUE.
      ITER = ITER + 1
      PRINT 1004, ITER
 1004 FORMAT(1HS,I4)
      DO 160 J = 1, NMIN1
      IF (I(J) . LE . I(J + 1)) GO TO 160
      OK = .FALSE.
      ITEMP = I(J)
      I(J) = I(J + 1)
      I(J + 1) = ITEMP
      TEMP = L(J)
      L(J) = L(J + 1)
      L(J + 1) = TEMP
      TEMP = A(J)
      A(J) = A(J + 1)
      A(J + 1) = TEMP
      TEMP = B(J)
      B(J) = B(J + 1)
      B(J + 1) = TEMP
  160 CONTINUE
      IF (.NOT . OK) GO TO 130
      DO 180 J = 1, NCOLORS
      I1 = I(J) / 1000
      I2 = I(J) - 1000 * I1
      WRITE (1, 1002) I1, I2, L(J), A(J), B(J)
 1002 FORMAT(I5, 1H-, I3, F11.2, 2F10.2)
  180 CONTINUE
      STOP
      END
```

Appendix D. Computer program for selecting sample pairs for observers' evaluation (Program SIXWAYS)

Following is the derivation of the equations used in Program SIXWAYS to determine if a given sample lay no more than 0.1 CIELAB unit from one of the six iso lines radiating from a given standard.

1. Distance between a point and an iso (hue-lightness) line

Let L_s, a_s, b_s be the L^*, a^*, b^* coordinates of the standard. The iso (hue-lightness) line is the straight line between the standard point and the achromatic axis at the L_s lightness level. Let L_p, a_p, b_p be the L^*, a^*, b^* coordinates of the point. The asterisks will be omitted for simplicity.

First, raise or lower the point vertically until it has the same lightness value as the standard. The distance moved in color space is $L_s - L_p$.

Now drop a perpendicular from the new position of the point to the iso (hue-lightness) line. The equation of the iso (hue-lightness) line in the constant L plane is

$$b = (b_s/a_s)a. \tag{D-1}$$

The equation of the perpendicular from the new position of the point is

$$(b - b_p)/(a - a_p) = -(a_s/b_s). \tag{D-2}$$

We solve Equations D-1 and D-2 simultaneously to get the point of intersection, which we will call (a_i, b_i). We find that

$$a_i = a_s\,(a_p\,a_s + b_p\,b_s)\ /\ (a_s^2 + b_s^2), \tag{D-3}$$

$$b_i = b_s\,(a_p\,a_s + b_p\,b_s)\ /\ (a_s^2 + b_s^2). \tag{D-4}$$

Now the distance between the point and the iso (hue-lightness) line, which we will call D_1, is

$$D_1 = \left[(L_s - L_p)^2 + (a_p - a_i)^2 + b_p - b_i)^2\right]^{\frac{1}{2}}. \tag{D-5}$$

Substituting Equations D-3 and D-4 into Equation D-5, we obtain

$$D_1 = \left[(L_s - L_p)^2 + (a_s\, b_p - a_p\, b_s)^2 \Big/ (a_s^2 + b_s^2) \right]^{\frac{1}{2}}. \qquad \text{(D-6)}$$

2. Distance between a point and an iso (chroma-lightness) line

We first raise or lower the point vertically until it has the same lightness value as the standard, as we did under 1.

We then drop a perpendicular from the point to the iso (hue-chroma) line. The equation of the iso (hue-chroma) line is

$$(b - b_s) \;/\; (a - a_s) = -(a_s / b_s). \qquad \text{(D-7)}$$

The equation of the perpendicular from the new position of the point is

$$(b - b_p) \;/\; (a - a_p) = b_s / a_s. \qquad \text{(D-8)}$$

We solve Equations D-7 and D-8 simultaneously to get the point of intersection, which we will call (a_i, b_i). We find that

$$a_i = \left[a_s\, (a_s^2 + b_s^2) + b_s\, (a_p\, b_s - a_s\, b_p) \right] \Big/ (a_s^2 + b_s^2) \qquad \text{(D-9)}$$

$$b_i = \left[b_s\, (a_s^2 + b_s^2) - a_s\, (a_p\, b_s - a_s\, b_p) \right] \Big/ (a_s^2 + b_s^2) \qquad \text{(D-10)}$$

Substituting Equations D-9 and D-10 into Equation D-5 (with D_2 substituted for D_1) gives

$$D_2 = \left[(L_s - L_p)^2 + (a_s^2 + b_s^2 - a_p\, a_s - b_p\, b_s)^2 \Big/ (a_s^2 + b_s^2) \right]^{\frac{1}{2}} \qquad \text{D-11)}$$

3. Distance between a point and an iso (hue-chroma) line

The required distance is

$$D_3 = \left[(a_s - a_p)^2 + (b_s - b_p)^2 \right]^{\frac{1}{2}} \qquad \text{(D-12)}$$

```
      PROGRAM SIXWAYS (INPUT, OUTPUT, TAPE1, TAPE7)
C
C         IN A GROUP OF COLOR POINTS IN CIELAB SPACE, DETERMINES
C         ALL COLORS THAT ARE ON ISO(HUE-LIGHTNESS),
C         ISO(HUE-CHROMA) AND ISO(CHROMA-LIGHTNESS)
C         LINES RADIATING FROM EACH POINT IN TURN.
C
      DIMENSION A(400), B(400), NUMBER(400), NUMB(40, 3, 2),
     +DE(40, 3, 2), NP(3, 2)
C
      REAL L(400)
C
C         READ FILE OF COLOR POINTS.
C
      J = 1
      REWIND 1
      REWIND 7
  100 READ (1, 1001) NUMBER(J), L(J), A(J), B(J)
      IF (NUMBER(J) . EQ . 7H9999999) GO TO 120
      J = J + 1
      GO TO 100
  120 NCOLORS = J - 1
C
C         SET UP DO LOOP BASED ON EACH POINT.
C
      DO 300 J = 1, NCOLORS
      PRINT 1014, J
      WRITE (7, 1002) NUMBER(J)
C
C         SET UP DO LOOP BASED ON THE THREE ISO LINES IN
C         COLOR SPACE.
C
      DO 220 IVIA = 1, 3
      M1 = 0
      M2 = 0
C
C         SET UP DO LOOP BASED ON TESTING EACH POINT TO SEE
C         IF IT IS ON THE ISO LINE.
C
      DO 200 K = 1, NCOLORS
      IF (J . EQ . K) GO TO 200
      GO TO (130, 135, 140), IVIA
C
C         FORMULAS FOR DISTANCE BETWEEN COLOR POINT AND ISO LINE.
C
  130 DIST = SQRT((L(J) - L(K)) ** 2 + (A(J) * B(K) - A(K) * B(J))
     + ** 2 / (A(J) ** 2 + B(J) ** 2))
      GO TO 150
  135 DIST = SQRT((L(J) - L(K)) ** 2 + (A(J) ** 2 + B(J) ** 2
     + - A(K) * A(J) - B(K) * B(J)) ** 2 / (A(J) ** 2 + B(J) ** 2))
      GO TO 150
  140 DIST = SQRT((A(J) - A(K)) ** 2 + (B(J) - B(K)) ** 2)
C
C         IF DISTANCE IS TOO GREAT, ELIMINATE THE POINT.
C
  150 IF (DIST . GT . 0.1) GO TO 200
C
C         DETERMINE ON WHICH SIDE OF THE ISO LINE THE POINT LIES.
C         CALCULATE THE COLOR DIFFERENCE BETWEEN THE POINT
C         AND THE REFERENCE COLOR.
C
      GO TO (160, 165, 170), IVIA
  160 BI = B(J) * (A(K) * A(J) + B(K) * B(J)) / (A(J) ** 2 + B(J)
```

```
     + ** 2)
      IF (BI . GT . B(J)) GO TO 180
      GO TO 190
  165 AI = A(J) + B(J) * (A(K) * B(J) - A(J) * B(K)) / (A(J) ** 2
     + + B(J) ** 2)
      IF (AI . GT . A(J)) GO TO 190
      GO TO 180
  170 IF (L(K) . GT . L(J)) GO TO 180
      GO TO 190
  180 M1 = M1 + 1
      NUMB(M1, IVIA, 1) = NUMBER(K)
      DE(M1, IVIA, 1) = SQRT((L(J) - L(K)) ** 2 + (A(J) - A(K))
     + ** 2 + (B(J) - B(K)) ** 2)
      GO TO 200
  190 M2 = M2 + 1
      NUMB(M2, IVIA, 2) = NUMBER(K)
      DE(M2, IVIA, 2) = SQRT((L(J) - L(K)) ** 2 + (A(J) - A(K))
     + ** 2 + (B(J) - B(K)) ** 2)
  200 CONTINUE
      NP(IVIA, 1) = M1
      NP(IVIA, 2) = M2
  220 CONTINUE
C
C         PRINT OUT RESULTS FOR EACH STANDARD POINT.
C
      NMAX = MAX0(NP(1, 1), NP(1, 2), NP(2, 1), NP(2, 2), NP(3, 1),
     + NP(3, 2))
      IF (NMAX . EQ . 0) GO TO 270
      WRITE (7, 1011)
      DO 250 K = 1, NMAX
      WRITE (7, 1004)
      DO 250 M1 = 1, 3
      DO 250 M2 = 1, 2
      IF (K . GT . NP(M1, M2)) GO TO 250
      IF (M1 . EQ . 1 . AND . M2 . EQ . 1) WRITE (7, 1005)
     + NUMB(K, M1, M2), DE(K, M1, M2)
      IF (M1 . EQ . 1 . AND . M2 . EQ . 2) WRITE (7, 1006)
     + NUMB(K, M1, M2), DE(K, M1, M2)
      IF (M1 . EQ . 2 . AND . M2 . EQ . 1) WRITE (7, 1007)
     + NUMB(K, M1, M2), DE(K, M1, M2)
      IF (M1 . EQ . 2 . AND . M2 . EQ . 2) WRITE (7, 1008)
     + NUMB(K, M1, M2), DE(K, M1 ,M2)
      IF (M1 . EQ . 3 . AND . M2 . EQ . 1) WRITE (7, 1009)
     + NUMB(K, M1 ,M2), DE(K, M1 ,M2)
      IF (M1 . EQ . 3 . AND . M2 . EQ . 2) WRITE (7, 1010)
     + NUMB(K, M1 ,M2), DE(K, M1, M2)
  250 CONTINUE
      GO TO 300
  270 WRITE (7, 1012)
  300 CONTINUE
      STOP
C
C         FORMAT STATEMENTS
C
 1001 FORMAT(2X,A7,F12.2,2F10.2)
 1002 FORMAT(////* SAMPLE NUMBER *, A7)
 1003 FORMAT(I5)
 1004 FORMAT(* *)
 1005 FORMAT(1H+,    A7,F6.2)
 1006 FORMAT(1H+,17X,A7,F6.2)
 1007 FORMAT(1H+,34X,A7,F6.2)
 1008 FORMAT(1H+,51X,A7,F6.2)
 1009 FORMAT(1H+,68X,A7,F6.2)
 1010 FORMAT(1H+,85X,A7,F6.2)
 1011 FORMAT(*  PLUS CHROMA*, 5X, *MINUS CHROMA*, 7X,
     +*PLUS HUE*, 9X, *MINUS HUE*, 7X, *PLUS LIGHT*, 7X,
```

```
     +*MINUS LIGHT*)
 1012 FORMAT(* NO POINTS FOUND IN ANY DIRECTION*)
 1014 FORMAT(1H≤,I5)
C
      END
```

Typical portion of output from Program Sideways

SAMPLE NUMBER 519-152

PLUS CHROMA	MINUS CHROMA	PLUS HUE	MINUS HUE	PLUS LIGHT	MINUS LIGHT
	513- 10 .21	528-610 .09	967-140 .06	519-336 .46	528-610 .09
	528-610 .09	528-642 .16		526-753 2.08	967-140 .06
	967-140 .06	536-835 .12		528-574 .15	
				529-757 .29	
				529-765 .31	
				529-779 .61	

SAMPLE NUMBER 519-176

PLUS CHROMA	MINUS CHROMA	PLUS HUE	MINUS HUE	PLUS LIGHT	MINUS LIGHT
513- 30 .13	526- 40 .46	513- 30 .13	526-108 .11	51-989*00.20	526-123 .11
526-135 .11	526-123 .11	526-135 .11		511-167 1.08	526-135 .11
528-610 .18	528-584 .10	528-584 .10		512-496 .74	
	531-446 .29			513- 9 .19	
				513- 10 .17	
				526- 71 .56	
				526-748 1.66	
				526-748 1.68	
				526-753 1.49	
				528-607 .75	
				528-636 .82	
				529-776 .74	
				530-345 .87	

SAMPLE NUMBER 519-223

PLUS CHROMA	MINUS CHROMA	PLUS HUE	MINUS HUE	PLUS LIGHT	MINUS LIGHT
	513- 9 .23	529-748 .05	528-574 .10	526-753 1.96	536-835 .18
	513- 10 .29		536-835 .10	528-574 .10	
	526-165 .44			529-748 .05	
	528-114 .20			529-757 .18	
	528-574 .10			529-765 .19	
	529-748 .05			529-779 .49	
	530-351 .61				
	534-980 .68				
	536-834 .24				
	536-835 .10				
	536-929 .47				

Appendix E. Computer program for randomizing order of presentation of sample pairs (Program FUNFER)

```
      PROGRAM FUNFER (INPUT, OUTPUT, TAPE1)
C
C         PREPARES RANDOMIZED LIST OF COMPARISONS FOR THE INSPECTOR
C
      DIMENSION NAME (2, 24), ITAL(24)
C
      LOGICAL DONE
C
      DATA NAME/
     +1001,8444,8444,1001,7030,7430,7430,7030,
     +7291,8443,8443,7291,7052,7000,7000,7052,
     +0709,7023,7023,0709,0847,0979,0979,0847,
     +0866,0650,0650,0866,0006,0837,0837,0006,
     +0681,0984,0984,0681,0714,0644,0644,0714,
     +5000,5327,5327,5000,1000,5310,5310,1000/
C
      DO 100 J = 1, 24
  100 ITAL(J) = 0
  150 K = IFIX(RANF(0.0) * 24. + 1.)
      IF (ITAL(K) . GE . 10) GO TO 150
      WRITE (1, 1001) (NAME(J, K), J = 1, 2)
      ITAL(K) = ITAL(K) + 1
      DONE = .TRUE.
      DO 200 J = 1, 24
      IF (ITAL(J) . LT . 10) DONE = .FALSE.
  200 CONTINUE
      IF (DONE) STOP
      GO TO 150
 1001 FORMAT(//1X, I4, I9)
      END
```

```
****** 17.25.57.    MANNYLW 000081 LINES PRINTED   /// END OF LIST ///   LQ  22
```



```
****** 17.25.57.    MANNYLW 000081 LINES PRINTED   /// END OF LIST ///   LQ  22
```

Appendix F. Computer program for calculation of 50% acceptability limits by logistic function, (Program LOGIT).

The logistic function was used by Berkson (4, 5) to establish the dose of a drug that is lethal to 50% of the population exposed, the so-called L.D.50 value. The use in our work is analogous: we wish to determine the color difference at which a given inspector will pass examples 50% of the time. The papers of Berkson should be consulted for details of the function and its use; we will just present the equations here.

The logistic function is

$$P = 1 - Q = \frac{1}{1 + e^{-(\alpha + \beta x)}} \tag{F-1}$$

In this equation, x is an independent variable. In Berkson's work with drugs, it is the logarithm of the dosage; in our work, following Berkson, we used the logarithm of the color difference. The symbol P represents, in Berkson's work, the calculated fraction of test animals killed by the dose corresponding to x; in our work it is the calculated fraction of times the inspector passes the pair of samples having a color difference corresponding to x. The quantities α and β are parameters.

Equation F-1 can be linearized by using the function ln (P/Q), which is equal to $\alpha + \beta x$. This function is known as the logit of P. Thus, plotting the logit of P against x should yield a straight line, if the assumptions on which the logistic function is based are valid.

4 J. Berkson, "Application of the Logistic Function to Bio-Assay," Am. Stat. Assn. J., 39, 357-365 (1944).

5. J. Berkson, "A Statistically Precise and Relatively Simple Method of Estimating the Bio-Assay with Quantal Response, Based on the Logistic Function," Am. Stat. Assn. J., 48, 565-599 (1953).

The following analysis applies to one inspector looking at a series of four color differences, where each color difference is shown to him 10 times. Let p_i be the <u>observed</u> fraction of passes at the color difference corresponding to x_i (remember that P was the <u>theoretical</u> fraction of passes), $q_i = 1 - p_i$, and $l_i = \ln (p_i/q_i)$. It can then be shown that good estimates of α and β (denoted by a and b) can be obtained by the following normal equations, where the summations are taken over the four color differences:

$$a = \frac{\Sigma pql \; \Sigma pqx^2 - \Sigma pqlx \; \Sigma pqx}{\Sigma pq \; \Sigma pqx^2 - (\Sigma pqx)^2} , \tag{F-2}$$

$$b = \frac{\Sigma pq \; \Sigma pqlx - \Sigma pqx \; \Sigma pql}{\Sigma pq \; \Sigma pqx^2 - (\Sigma pqx)^2} . \tag{F-3}$$

Once a and b have been calculated, the x value corresponding to a 50% pass is given very simply. Since $\ln (P/Q) = \alpha + \beta x$, if $P = Q = 0.5$ then $\ln (P/Q) = 0$ and $x_{50} = -(\alpha/\beta)$. We thus have

$$\Delta E_{50} = e^{-(a/b)} \tag{F-4}$$

since x represented the logarithm of the color difference. The symbol ΔE_{50} refers to the color difference for which the inspector will pass the sample 50% of the time.

The standard deviation of the ΔE_{50} value is calculated by the following equations:

$$s_{a'}^2 = 1/(10\ \Sigma\ pq), \tag{F-5}$$

$$s_b^2 = 1/\left[10\ \Sigma\ pq\ (x - \bar{x})^2\right], \tag{F-6}$$

where $\bar{x}$ is the x value averaged over the four color differences.

$$s_{x_{50}}^2 = (1/b^2)\left[s_{a'}^2 + s_b^2\ (x_{50} - \bar{x})^2\right], \tag{F-7}$$

$$s_{\Delta E_{50}} = \Delta E_{50} s_{x_{50}}, \tag{F-8}$$

where $s_{\Delta E_{50}}$ is the desired standard deviation.

In entering the observers' data for fraction of passes, we avoided entering 1 for 100% passes or 0 for 0% passes, since the corresponding logits would have been infinite. Following Berkson, we used 0.95 and 0.05, respectively.

```
      PROGRAM LOGIT (INPUT, OUTPUT, TAPE1)
C
C          COMPUTES 50 PER CENT ELLIPSOID BOUNDARIES
C          BY THE METHOD OF LOGISTIC FUNCTIONS.
C
      DIMENSION DE(4, 6, 2), Q(4), IQ(4, 3), W(4), D50(3),
     +STDEV(3)
C
      REAL LOGIT(4), LOGDE(4)
C
      INTEGER CCHHLL, GUY, SIZE, COLOR
C
      DATA DE/.4, .8, 1.13, 1.52, .4, .8, 1.13, 1.52, .4, .77,
     +1.17, 1.32, .4, .77, 1.17, 1.45, .6, 1.17, 1.82, 2.29,
     +.6, 1.17, 1.82, 2.29, .5, .92, 1.33, 1.92, .5, .92, 1.33,
     +1.92, .25, .47, .69, 1.01, .25, .48, .69, 1.01, .66, 1.32,
     +1.95, 2.65, .66, 1.32, 1.95, 2.65/
C
C          DO LOOPS FOR COLOR STANDARD, DIRECTION IN
C          COLOR SPACE, AND INSPECTOR.
C
      REWIND 1
      DO 300 COLOR = 1, 2
      IF (COLOR . EQ . 1) PRINT 1003
      IF (COLOR . EQ . 2) PRINT 1004
      PRINT 1012
      DO 300 CCHHLL = 1, 6
      GO TO (101, 102, 103, 104, 105, 106), CCHHLL
  101 PRINT 1005
      GO TO 110
  102 PRINT 1006
      GO TO 110
  103 PRINT 1007
      GO TO 110
  104 PRINT 1008
      GO TO 110
  105 PRINT 1009
      GO TO 110
  106 PRINT 1010
  110 AVX = 0.
      DO 120 SIZE = 1, 4
      LOGDE(SIZE) = ALOG(DE(SIZE, CCHHLL, COLOR))
  120 AVX = AVX + LOGDE(SIZE)
      AVX = 0.25 * AVX
      DO 250 GUY = 1, 3
C
C          READ PERCENT PASSES.
C
      READ (1, 1002) (Q(SIZE), SIZE = 1, 4)
      DO 130 SIZE = 1, 4
      IQ(SIZE, GUY) = 100. * Q(SIZE) + .0001
      IF (IQ(SIZE, GUY) . GT . 90) IQ(SIZE, GUY) = 100
      IF (IQ(SIZE, GUY) . LT . 10) IQ(SIZE, GUY) = 0
  130 CONTINUE
C
C          CALCULATION OF LOGISTIC FUNCTIONS AND WEIGHTS.
C
      DO 150 SIZE = 1, 4
      P = 1. - Q(SIZE)
      W(SIZE) = P * Q(SIZE)
  150 LOGIT(SIZE) = ALOG(P / Q(SIZE))
C
C          CALCULATION OF SUMMATION VALUES.
```

```
C
      SUMW = 0.
      SUMWL = 0.
      SUMWLX = 0.
      SUMWX = 0.
      SUMWX2 = 0.
      SUMWXX2 = 0.
      DO 200 SIZE = 1, 4
      SUMW = SUMW + W(SIZE)
      SUMWL = SUMWL + W(SIZE) * LOGIT(SIZE)
      SUMWLX = SUMWLX + W(SIZE) * LOGIT(SIZE) * LOGDE(SIZE)
      SUMWX = SUMWX + W(SIZE) * LOGDE(SIZE)
      SUMWX2 = SUMWX2 + W(SIZE) * LOGDE(SIZE) ** 2
  200 SUMWXX2 = SUMWXX2 + W(SIZE) * (LOGDE(SIZE) - AVX) ** 2
C
C         CALCULATION OF SLOPE AND INTERCEPT OF LEAST SQUARES LINE.
C
      DENOM = SUMW * SUMWX2 - SUMWX ** 2
      A = (SUMWL * SUMWX2 - SUMWLX * SUMWX) / DENOM
      B = (SUMW * SUMWLX - SUMWX * SUMWL) / DENOM
C
C         50 PER CENT POINT.
C
      X50 = -A / B
      D50(GUY) = EXP(X50)
C
C         STANDARD DEVIATION OF 50 PER CENT POINT.
C
      VARAPR = 1. / (10. * SUMW)
      VARB = 1. / (10. * SUMWXX2)
      VARX50 = (1. / B ** 2) * (VARAPR + VARB * (X50 - AVX) ** 2)
      VARD50 = VARX50 * D50(GUY) ** 2
      STDEV(GUY) = SQRT(VARD50)
  250 CONTINUE
C
C         PRINT RESULTS.
C
      PRINT 1011, (DE(SIZE, CCHHLL, COLOR), (IQ(SIZE, GUY),
     +GUY = 1, 3), SIZE = 1, 2)
      PRINT 1001, DE(3, CCHHLL, COLOR), (IQ(3, GUY),
     +D50(GUY), STDEV(GUY), GUY = 1, 3)
      PRINT 1011, DE(4, CCHHLL, COLOR),(IQ(4, GUY),
     +GUY = 1, 3)
  300 CONTINUE
      STOP
C
C         FORMAT STATEMENTS.
C
 1001 FORMAT(F5.2, I6, F7.2, 2H +/1H+, 17X, 2H -, F5.2,
     +I5, F7.2, 2H +/1H+, 36X, 2H -, F5.2, I5, F7.2, 2H +/
     +1H+, 55X, 2H -, F5.2)
 1002 FORMAT(F15.0)
 1003 FORMAT(////22X, *COLOR: OLIVE GREEN*)
 1004 FORMAT(////26X, *COLOR: TAN*)
 1005 FORMAT(//6X, *PLUS CHROMA*/)
 1006 FORMAT(//6X, *MINUS CHROMA*/)
 1007 FORMAT(//6X, *PLUS HUE*/)
 1008 FORMAT(//6X, *MINUS HUE*/)
 1009 FORMAT(//6X, *PLUS LIGHT*/)
 1010 FORMAT(//6X, *MINUS LIGHT*/)
 1011 FORMAT(F5.2, I6, 2I19/F5.2, I6, 2I19)
 1012 FORMAT(///* DELTA*, 7X, *BILL*, 16X, *MOE*, 15X, *CAROL*
     +/3X, *E*/4X, 3(4X, *PCT.*, 3X, *BOUNDARY*)/6X, 3(2X, *PASS*,
     +2X, *^ STD. DEV.*))
C
      END
```

Appendix G. Derivation and Meaning of the Acceptability Equation

If the acceptability equation, Equation 1, were to have the square root bracket removed, and if ΔA were set equal to unity, it would become an ellipsoid equation in which the center of the ellipsoid is placed at the standard point in CIELAB space, the three semiaxes are made equal in size to the tolerance values determined by the inspectors, and the angle θ is made equal to the hue angle of the standard. Thus, in Equation B-3, c, the chroma tolerance, replaces p; h, the hue tolerance replaces q; and v, the lightness tolerance, replaces r. The coordinates of the standard, a_0^*,b_0^* and L_0^*, replace those of the center of the ellipsoid, a_c^*, b_c^* and L_c^*. With these changes, Equations B-4 to B-8 become Equations 2 to 6, and Equation 1 modified as just indicated is equivalent to Equation B-9. This equation defines what we call the "acceptability ellipsoid."

Now if we substitute the coordinates of any point into this equation, we generally will no longer calculate unity but some other value that we have called S^2 in Appendix B and we will call $(\Delta A)^2$ here. We showed in Appendix B that we have defined a new ellipsoid that passes through the sample point in which the linear distances are expanded or contracted by a factor of S (or ΔA). We thus see the rationale of taking the square root of the expression in Equation 1; it is that we wish to calculate the factor (ΔA) by which the linear distance from the standard to the sample in question is greater or less than the linear distance along the same line but from the standard to the acceptability ellipsoid. If ΔA is greater than 1, the point lies outside of the acceptability ellipsoid; if it is less than 1, the point is inside the acceptability ellipsoid.